N° 22,620 A.D. 1908

Date of Application, 24th Oct., 1908—Accepted, 3rd June, 1909

COMPLETE SPECIFICATION.

A Logarithmic Disc Calculating Machine.

I, ERNST LEDER, of Hasenheide 54, Berlin in the German Empire, Manufacturer, do hereby declare the nature of this invention and in what manner the same is to be performed, to be particularly described and ascertained in and by the following statement:—

This invention relates to a logarithmic disc calculating machine and utilises in its construction an improved form of a logarithmic disc of which I am the inventor and for which I obtained Letters Patent in the German Empire No. 104927 dated 27th July 1897.

In order that my present invention may be clearly understood, as the original disc in question did not form subject matter for Letters Patent in Great Britain, I will proceed to describe briefly the simple form of the disc and its use for calculating with the aid of logarithms:—

I provide a circular disc of radius R (say for instance 15 cm). This disc is divided by n (say 100) radii into as many congruent sectors. A smaller circle is described concentric to the edge of the disc, which circle I shall call hereafter the inner circle bounding the scale, or simply the "inner circle." This circle is of a radius r (say 5 cm.) so that on each of the radii there is a portion cut off between the circumference of the disc and the inner circle of the length $R - r$ (in this case 10 cm.)

Opposite to the sectors thus bounded by the portions of the radii, the edge of the disc and the inner circle, and at the inner circle there are marked consecutively, proceeding in the direction of the motion of the hands of a clock, the numbers 00, 01, 02, . . .98, 99. . . $(n - 2)$, $(n - 1)$. Each of these numbers indicates how many of the beforementioned parts of radii, from the commencement of the scale set on the radii, precede that radius on which a particular number is marked. For instance, if the graduation on the scale lies on the radius marked with the number 13, at a distance of a mm. from the inner circle, then the length of the scale from the commencement to the same point is $a + 13 \times 100$ mm.

The total length of the n (here 100) parts of radii represents a length $n\ (R - r)$ or in the particular case given 100×10 cm.

This distance is taken as the unit of measurement, and consequently represents log 10.000 and then for every number lying between 1.000 and 10.000 a graduation is set on one of the n (100) parts of radii in such a way that a point, travelling from the commencement of the scale along all the radii preceding that on which a particular graduation is marked and along that radius as far as that graduation, would pass over a distance which would bear the same ratio to the whole length of the scale, viz. $n\ (R - r)$ as $\log z : \log 10.000 = \log z : 1$, where z indicates the number set against the graduation in question.

Log 2.000 = 0.3010. As the distance $n\ (R - r) = 100 \times 10$ cm. = 10.000 mm. represents log 10 = 1.0000, similarly corresponding to the log 2.000 = 0.3010 there is a length of $0.3010 \times 10{,}000$ mm. = 3010 mm. As each radial portion is 10 cm = 100 mm long, 3010 mm represents 30 whole radial pieces and 0.10 of the 31st; consequently the graduation mark for 2.000 lies on the 31st radial portion (which is marked at the inner circle by the number 30) at a distance of 10 mm from the inner circle. In a corresponding manner the graduation for every number lying between 1.000 and 10.000 is determined.

A disc constructed as just described can be employed by itself as a calculating

[*Price 8d.*]

apparatus. For instance, with its aid, the logarithm corresponding to each number and the number corresponding to each logarithm can be found.

Suppose it is required to find log 13.46 it is only necessary to find the graduation marked 1346 and to measure with a millimetre scale its distance from the inner circle. This is found to be 90 mm. This number 90 gives the third and fourth figures of the mantissa. The first and second are at the inner circle, viz 12. Consequently as according to the well-known rules of logarithms the characteristic is 1, the result is that log 13.46 is 1.1290.

Conversely, it is easy to find the logarithm of a number. Suppose $\log x = 2.3783$ and it is required to find x, the scale is placed against the sector marked with the number 37, and the number on the disc is read corresponding with the 83rd graduation of the scale. It is found to be 2390. Accordingly, remembering that the characteristic is 2, log 2.3783 is the logarithm of the number 239.

It will be observed that the graduations representing numbers the mantissæ of which agree as to the first two decimal figures lie on the same radius.

In carrying out multiplication and division, by the theory of logarithms, the logarithms of the factors must be added or the logarithm of the divisor subtracted from the logarithm of the dividend, as the case may be.

Suppose the numbers z_1 and z_2 are to be multiplied. Suppose the graduation for $\log z_1$ lies on the radius marked with the numeral a^1 at a distance x_1 from the inner circle, whilst the graduation for $\log z_2$ lies on the radius marked with the number a_2 at a distance x_2 from the inner circle.

Suppose the whole length of the scale to be denoted by L.

Then $L = n(R-r)$ [$= 100 \times 10$cm.]

By the foregoing definitions we have

$$\text{I. } \log z_1 = \frac{a_1\ (R - r) + x_1}{L}$$

$$\text{II. } \log z_2 = \frac{a_2\ (R - r) + x_2}{L} \text{ consequently}$$

$$\text{III. } \log (z_1 + z_2) = \log z_1 + \log z_2 = \frac{(a_1 + a_2)(R - r) + (x_1 + x_2)}{L}$$

Let the graduation mark for the logarithm of $z_1 + z_2$ lie on the radius a_3 at a distance x_3 from the inner circle then by the foregoing definition

$$\text{IV. } \log (z_1 + z_2) = \frac{a_3 (R - r) + x_3}{L}$$

From III. and IV. it follows

$$\text{V. } \frac{a_3 (R - r) + x_3}{L} = \frac{(a_1 + a_2)(R - r) + (x_1 + x_2)}{L}$$

or as a_3, a_1, a_2, are whole numbers, $a_3\,(R - r)$ and $(a_1 + a_2)\,(R - r)$ are multiples of entire portions of radii, but x_1, x_2 and x_3 are fractions of portions of radii, consequently

$$\text{VI } a_3 (R - r) = (a_1 + a_2)(R - r) \text{ or}$$
$$\text{VI}^1\ a_3 = a_1 + a_2.$$
$$\text{VII. } x_3 = x_1 + x_2.$$

Hence it follows :—

1. The number denoting the portion of radius on which the graduation mark for the product of two numbers lies is found by adding the numbers denoting the part radii on which the graduations corresponding to the factors lie.

2. The distance of the graduation for the product of two numbers from the inner circle is found by adding the distances of the graduations for the factors from the inner circle.

If the part radius is known on which a graduation must lie, and the distance at which it lies from the inner circle, its position is fully determined. The rules 1 and 2 given above are therefore sufficient to determine the position of the graduation for the product, when the positions of the graduations for the factors are known.

The following must be borne in mind:

$$\frac{x_1}{R - r} \text{ and } \frac{x_2}{R - r} \text{ are proper fractions.}$$

For their sum therefore one of the following relations holds good

$$0 < \frac{x_1 + x_2}{R - r} < 1$$

or

$$1 < \frac{x_1 + x_2}{R - r} < 2$$

The distance of the graduation for the product from the inner circle (x_3) is, therefore, in the second case greater than $R - r$, that is to say the graduation would fall beyond the edge of the disc, or in other words it lies on the following radius, and as far from the inner circle as it would be beyond the edge of the disc, that is to say at a distance $x_1 + x_2 - (R - r)$. In this case in determining the place occupied by the graduation of the product, the sum $a_3 = a_1 + a_2$ is raised by 1, and the sum

$$\frac{x_3}{R - r} = \frac{x_1 + x_2}{R - r}$$

diminished by 1.

To the logarithmic disc just described, in place of using a separate scale, I fitted a simple arrangement for carrying out the before described operations mechanically.

A scale for effecting the reading was mounted so as to rotate radially around the centre of the disc and in such a way that it could be utilised for determining the distances of graduations from the inner circle and adding distances by means of a slide. Again a disc was mounted so as to be rotatable, which was furnished with graduations by means of which the numbers set opposite the portions of radii could be added.

In utilising this calculating apparatus certain defects became apparent, all of which are obviated by the disc and mechanism forming the subject matter of the present specification.

In the simple disc, as the scale extended right up to the edge, it followed that each part radius $(R - r)$ and consequently each of the sections into which the scale was broken up joined immediately on to the preceding one. For example, the graduation for 1230 is on the radius marked 08, and that for 1231 on the radius marked 09, the consequence being with the old disc that part of the interval from 1230 to 1231 falls on one radius and part on another. If then it is required to make a calculation with the number 1230.5, that is to say to estimate the distance of the graduation for 1230.5 between 1230 and 1231, this estimation is difficult on account of the distribution of the interval between 1230 and 1231 in two parts, and consequently the work is inexact.

In the disc employed with my new machine an improvement is effected in this respect. In all cases where the interval between two successive graduations would fall partly on one and partly on the following radius, the remainder of the interval is represented as a prolongation of the first radius, so that an exact estimate can be made and consequently the correct position for the subordinate graduation between two given graduations is determined.

Again with the slide and scale for effecting readings on the part radii, and adding parts, and with the disc for effecting the addition *etc* of the part radii, difficulties were experienced, as although the work was largely mechanical, a good many points had to be borne in mind in bringing the slide or disc back to the zero points for each factor. In some cases, moreover, it happened that the graduation to which the rotating disc was to be set was covered by the slide or slide carrier so that the setting of the disc could not be properly effected or could only be effected with difficulty.

In the present machine all this is obviated by the use of improved scales and slides and by the addition of special counting apparatus for the first two decimal figures of the mantissa.

In the accompanying drawings:—

Fig 1 is a top plan of the calculating apparatus;

Fig 2 is a vertical section on the line A, A Fig 1;

Fig 3 is a vertical section along the line B, B, Fig 1;

Fig 4 is a vertical section on the line C, C Fig 1;

Fig 5 shews the scale disc of the apparatus;

Fig 6 is a view of the registering disc for the first two decimal figures of the mantissa.

The same letters of reference are employed to denote the same parts in all the views:—

a is the disc bearing the scale. This disc is mounted on an axle *b*. This axle *b* is so mounted in the cylindrical casing of the apparatus that its boss coincides with that of the casing. With the aid of the milled head *c*, the axle *b* and with it the attached scale disc *a* can be rotated. The cover plate *d* of the apparatus is formed with a slit *e* of such shape and size that of the 100 radial portions of the scale marked on the scale disc only one is visible at one time through such slit, the other 99 being concealed by the cover plate *d*. On the cover plate *d* is fitted the slide carrier *f*, one edge of which corresponds with the portion of the scale visible in the slit. Parallel to this edge, in a groove *g* is fitted a movable slide *h*. Again, there is fitted parallel to the edge and on the sloping edge of the slide carrier *f* a measuring scale *i* which is divided into 100 equal parts, its length being equal to the one hundredth part of the length of the whole scale.

There is attached to the slide, so as to be capable of moving therein, the movable runner *k*, consisting of a frame *l* with a glass plate *m* set therein. This carries an etched line o^1 the direction of which falls in one plane with the pointer *o* attached to the frame *l*, which is placed normal to the direction of movement of the slide *h*. On the upper surface of the slide *h* are four graduations marked respectively + 2, + 1 0, — 1. These graduations are separated from one another by a distance equal to the one hundredth of the whole length of the scale and their direction is parallel to the direction o, o^1.

Under the cover plate is fitted the disc *q* which carries circumferentially around it the numbers 00 to 99, see Fig 6. These numbers are all concealed with the exception of one which comes under the opening *r* and is consequently visible to the person using the apparatus. The disc *q* is rotated by means of the milled nut *s*, which is attached to the axle *t*. On the same axle is a pinion *u* which gears with the toothed wheel *v*, the ratio of the teeth being as 1 to 20. The rotation of the toothed wheel *v* is indicated by the pointer *w* on the figured dial *x*. The pointer *w* is only mounted loosely on the axle of the wheel *v*, and is moved by the same by friction only. It can consequently when it is required to set it to any required position be moved without turning the wheel *v*.

y is another opening through which at all times one of the numbers 00 to 99 at the edge of the disc *a* is visible. It will be observed that these numbers are placed around the edge of the scale disc *a* consecutively. The number 1 at the edge of the disc is 90 degrees in advance of the first part radius indicated on the inner scale by the number 1, and so on. This arrangement is an arbitrary one, the reason that the numbers start 90 degrees from the numbers at the inside being merely to suit the construction of the particular apparatus, the idea being that when any particular scale arc is visible within the aperture *b* the number of that scale arc is denoted in the aperture *y*. *p* is a stop pin on the slide carrier *f*. *z* is an auxiliary scale the use of which will be hereinafter explained.

The details of construction of the calculating apparatus having been described I will now proceed to examine its operations with the aid of examples.

1. Required log 0.05736.

The disc *a* is turned until the graduation mark 5736 appears and then the slide *h* is slidden along until the pointer points to this graduation. In the opening *y* there appears 75. The etched line o^1 and pointer *o* correspond to the graduation 86.1 on the graduated scale *i*, therefore as the characteristic is minus 2, log 0.05736 = $\bar{2}$.75861.

2. Required to find the number the log of which is 2.3583.

The disc *a* is turned until 35 appears in the opening *y*, the slide *h* is slidden until the etched line o^1 and pointer *o* correspond with the graduation 83 of the graduated scale *i*, when the pointer points to 22819. As the characteristic is 2 the result follows that the number the logarithm of which is 2.3583 = 228.19.

3. Required $(25.4)^2$

The disc *a* is turned until 25.4 appears at the slit *e* and the slide *h* is slidden along until the pointer points to the same graduation. At the opening *y*, 40 appears. The etched stroke o^1 points to 48.3 on the graduated scale *i*. Therefore, log 25.4 = 1.40483.

In order to square a number its logarithm must be doubled. To avoid having to make any calculation, use is made of the auxiliary scale *z* to effect this doubling. The graduation 48.3 on the lower scale corresponds with the graduation 96.6 on the middle scale, consequently 2 × 48.3 = 96.6. The graduation 40 on the lower scale corresponds to 80 on the middle scale. Hence we find

$$\log (25.4)^2 = 2{\cdot}80966.$$

The slide is now slidden until the etched line o^1 and pointer *o* point at 96.6 of the graduated scale *i*, and the disc *a* is turned until in the opening *y* the number 80 appears. The pointer points to 64516, consequently, as the characteristic is 2 we have

$$(25.4)^2 = 645.16.$$

4. Required $(17.9)^3$.

The disc *a* is turned until 179 appears, when 25 is seen in the opening *y*. The slide *h* is slidden until the pointer points to 179, and then the etched mark o^1 and pointer *o* correspond to 28.5 of the graduated scale *i*. By the help of the auxiliary scale *z*, the logarithm is multiplied by 3. In the middle scale there appears opposite 28.5 on the upper scale 85.5 and opposite 25 on the upper scale 75. Therefore log $(17.9)^3$ = 3.75855.

The scale *a* is turned until 75 appears in the opening *y* and the slide *h* is slidden along until the etched marking o^1 and pointer *o* are at 85.5, when the pointer points to 5735.2, consequently, as the characteristic is 3 we have

$$(17.9)^3 = 5735.2.$$

5. Required $(1.0375)^{20}$.

The disc *a* is turned until 1.0375 appears, and the slide *h* is slidden into position with the pointer at this graduation. Then there appears in the opening *y*, 01, and the etched marking o^1 and pointer *o* correspond to 59.9 of the graduated scale *i*. Consequently log 1.0375 = 0.01599. Multiplying by 20 we have .3198. Consequently log $(1.0375)^{20}$ = 0.3198.

The disc *a* is turned until 32 appears and the slide *h* is slidden so that the first stroke of the graduated scale *i* correspond to the etched marking o^1 and pointer *o*. The pointer then points to 20883. Consequently we have

$$(1.0375)^{20} = 2.0883.$$

6. Required $\sqrt{7225}$.

The scale *a* is turned until 7225 appears in the opening *e*. At the opening *y*, 85 appears. The slide *h* is slidden along until the pointer points to 7225 and then the etched stroke o^1 and pointer o correspond to the stroke 88.4 on the graduated scale i. Therefore log 7225 = 3.85884.

Hence it follows that $\log \sqrt{7225} = \frac{1}{2}(3.85884)$.

The characteristic is found to be $\frac{1}{2} \times 3 = 1$ with remainder 1, and by the use of the auxiliary scale z, $\frac{1}{2}(185) = 92$ with remainder 1.

$\frac{1}{2}(188.4) = 94.2$.

Consequently $\log \sqrt{7225} = 1.92942$.

The disc is turned until 92 appears in the opening y, and the slide h is slidden along until the etched stroke o^1 and pointer o coincide with 94.2 of the graduated scale i, then the pointer points to 8500 so that taking the characteristic into account

$\sqrt{7225} = 85$.

7. Required $\sqrt[3]{4913}$.

The disc a is turned until 4913 appears in the slit e, and the slide h is slidden along until the pointer points to this graduation. In the opening y, 69 appears. The etched marking o^1 and pointer o correspond to 13.5 on the graduated scale i. Therefore log 4913 = 3.69135; but $\log \sqrt[3]{4913} = \frac{4913}{3} = \frac{3.69135}{3}$. The divisions $\frac{69}{3}$ and $\frac{135}{3}$ are carried out as follows with the aid of the auxiliary scale z.

Over graduation 69 of the middle scale appears 23 on the upper scale and over 13.5 of the middle scale there appears 4.5 on the upper scale.

Therefore $\log \sqrt[3]{4913} = 1.23045$.

The disc a is turned until 23 appears in the opening y and the slide h is slidden along until the etched marking o^1 and pointer o appear over 04.5 of the graduated scale i. The pointer points then to 1700. Consequently having regard to the characteristic 1 we have

$$\sqrt[3]{4913} = 17.$$

8. Required $\sqrt[7]{27.164}$.

The disc a is turned until 27164 appears and the slide h is slidden along until the pointer points to this graduation. There then appears in the opening y the number 43 and the etched stroke o^1 and pointer o coincide with 39.9 of the graduated scale i. Consequently log 27.164 = 1.43399. Hence it follows that

$$\log \sqrt[7]{27.164} = \log \frac{27.164}{7} = \frac{1.43399}{7} = 0.20486.$$

The disc a is turned until 20 appears in the opening y, and the slide h is slidden along until the etched stroke o^1 and pointer o coincide with 48.6, when the pointer points to 16027, consequently bearing the characteristic o in mind we have as the result

$\sqrt[7]{27.164} = 1.6027$.

9. Required the product of

$7.892 \times 89.23 \times 0.09234 \times 0.0002345 \times 3456.7 \times 0.6789$.

According to the rule for finding the characteristics the characteristics of the factors taken in order are 0, 1, −2, −4, 3, −1, therefore the characteristic of the result is −3. The hand w is set to minus 3 on the dial x to shew the characteristic.

The disc a is turned until 7892 appears and the slide h is slidden along until the pointer points to this graduation. The runner k is set back to the start against the stop pin p, and the milled head s is turned until at the opening r there appears the same number as in the opening y, that is to say 89. The disc a is turned until 8923 appears and the slide h is slidden until the indicator points to the corresponding graduation, the runner k is set back to the commencement and the milled head s

turned until at the opening *r* there appears the number 84, that is to say the last two numbers of the sum of the numbers in the two apertures, (89 + 95) = (1)84.

The disc *a* is turned until 9234 appears and the slide *h* is slidden along until the indicator points to this graduation. The runner *k* is set back to the commencement and the milled head *s* is turned until at the opening *r* the number 80 appears, that is the last two figures of the sum of the two numbers appearing at the opening 84 + 96 = (1) 80.

The disc *a* is turned until 2345 appears, and the slide *h* is slidden along until the indicator points to this graduation. The runner *k* is set back to the commencement and the milled head *s* turned until in the opening *r* the number 17 appears, that is the two last numbers of the sum of the numbers appearing in the two openings namely 80 + 37 = (1) 17.

The disc *a* is turned until 34567 appears, the slide *h* is slidden along until the indicator points to this graduation. The runner *k* is set back to the commencement and the milled head *s* turned until in the opening *r* the number 70 appears that is the sum of the numbers appearing in the two openings namely 17+53=70.

The disc *a* is turned until 6789 appears and the slide *h* is slidden along until the indicator points at this graduation. The runner *k* is set back to the commencement and the milled head *s* turned until in the opening *r* the number 53 appears, namely the last two figures of the sum of the numbers appearing in the two openings 70+83=(1) 53.

The runner *k* is pushed along to the marking plus 2 on the slide *h* which now appears between 00 and 100 on the graduated scale *i* and then the disc *a* turned so that 53+2=55 appears in the opening *y* then the pointer of the runner *k* shews 35785. As the characteristic indicated by the hand *w* is between +1 and + 2 the characteristic is 1 and the result of the whole calculation is 35.785.

10. Divide 355 by 113.

Turn the disc *a* until 355 appears and slide the slide *h* along until the indicator points to the corresponding graduation. Turn the milled head *s* until in both the openings the same number appears namely 55.

Turn the disc *a* until 113 appears and push the runner *k* until the indicator points to this graduation and then push the slide *h* back until the etched marking o^1 and pointer *o* correspond with the 00 graduation on the scale *i*. The difference of the numbers in the two openings namely 55 − 5 = 50. This must be reduced by 1 to 49 because against the scale *i* − 1 appears on the slide. The etched marking o^1 is brought opposite this − 1 marking, and then the disc *a* turned until 49 appears in the opening *y* when the indicator points to the result, which is 3.3416.

It will be understood that the particular construction of the apparatus hereinbefore described is merely given by way of example as I may vary the construction to suit requirements. For instance, in some cases, in place of having a pointer *o* in the same line with the etched line o^1 on the glass plate *m*, I may replace the pointer by a second glass plate projecting over the scale *i* and provided with a second etched line falling in the same straight line with the etched line o^1 which second etched line performs the functions of the pointer *o*.

Again, the particular method of indicating the change in the characteristic herein described which has been adapted from that in use with circular slide rules may be varied to suit requirements, or one of the other methods already proposed for adding or subtracting the characteristics in connection with circular slide rules adapted thereto.

I may also in some cases vary the marking of the disc. For instance, I may break up the logarithmic scale and distribute it over more than 100 radii making suitable alterations in the positions of the numbers, etc.

Again I may vary the graduations on the slides *etc.* to suit the requirements of any particular case.

Having now particularly described and ascertained the nature of my said invention and in what manner the same is to be performed I declare that what I claim is :—

1. A logarithmic disc calculating apparatus constructed and operating substantially as set forth.

2. A logarithmic calculating disc in which the logarithmic scale is spread over any required number of part radii and wherein the first division on each part radius is repeated on a prolongation of the preceding radius substantially as and for the purposes set forth.

3. A logarithmic disc calculating machine comprising a disc with the logarithmic scale spread over any desired number of part radii in combination with a sliding scale and with a runner on such scale and movable therewith and thereon substantially as and for the purposes set forth.

4. A logarithmic disc calculating machine comprising a disc with the logarithmic scale spread over any desired number of part radii, means for shewing in a suitable aperture the first two decimal figures of the mantissa and means for shewing such numbers or their sum in a second aperture all for the purposes set forth.

5. The combination with a machine of the class claimed in the preceding claims of mechanism for indicating by means of a pointer on a suitable dial the change of the characteristic when the mantissa passes from 0.99 to 1.00 or the logarithm from $a + 0.99$ to $a + 1$ and conversely.

6. A logarithmic disc calculating machine of the class described provided with an auxiliary scale substantially as and for the purpose set forth.

Dated this 24th day of October 1908.

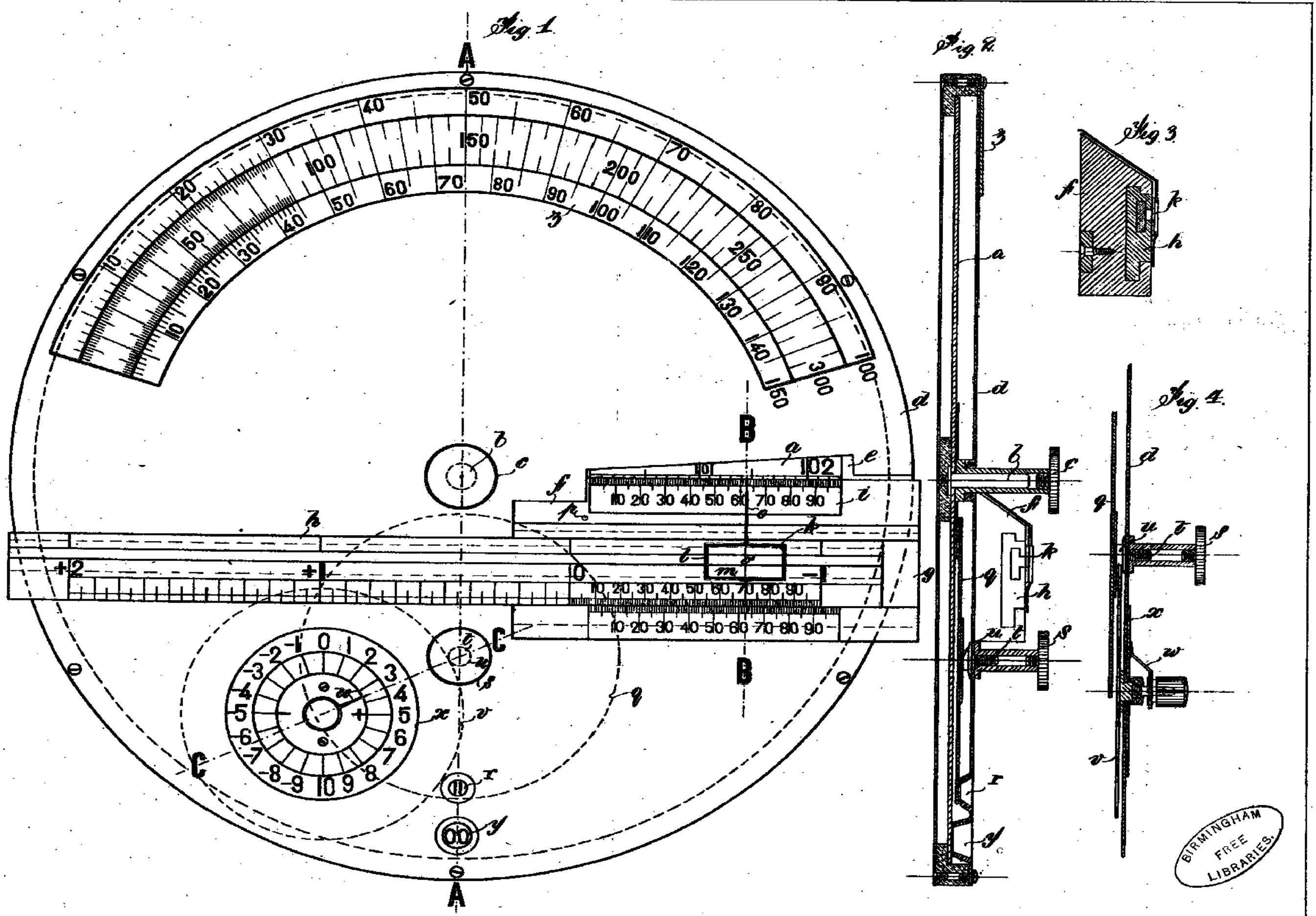
BROWNE & Co.,
Agents for the Applicant,
9, Warwick Court, Gray's Inn, W.C.

Redhill: Printed for His Majesty's Stationery Office, by Love & Malcomson, Ltd.—1909.

A.D. 1908. Oct. 24. N° 22,620.
LEDER'S Complete Specification.

(2 SHEETS)
SHEET 1.

[This Drawing is a reproduction of the Original on a reduced scale.]



Malby&Sons, Photo-Litho.

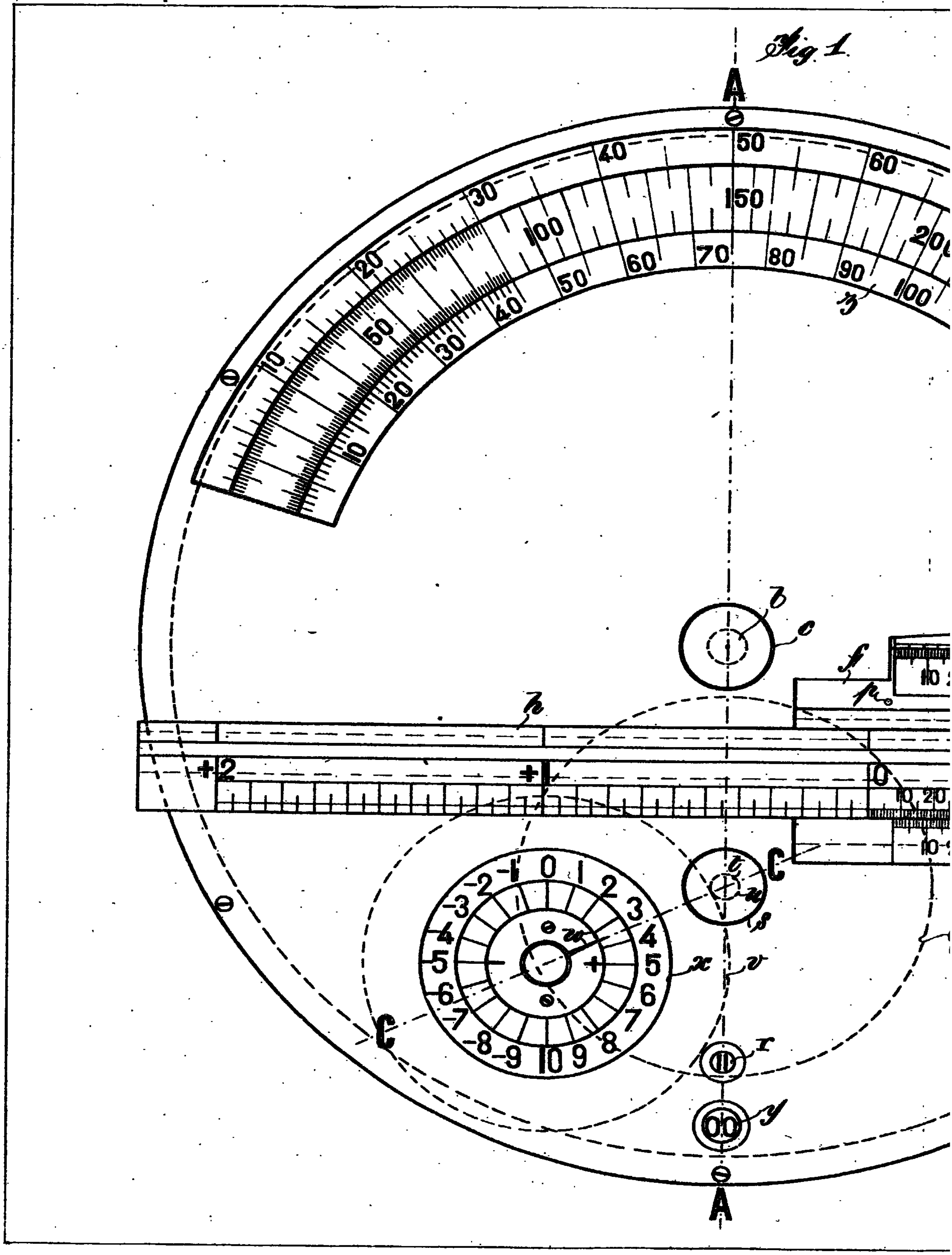
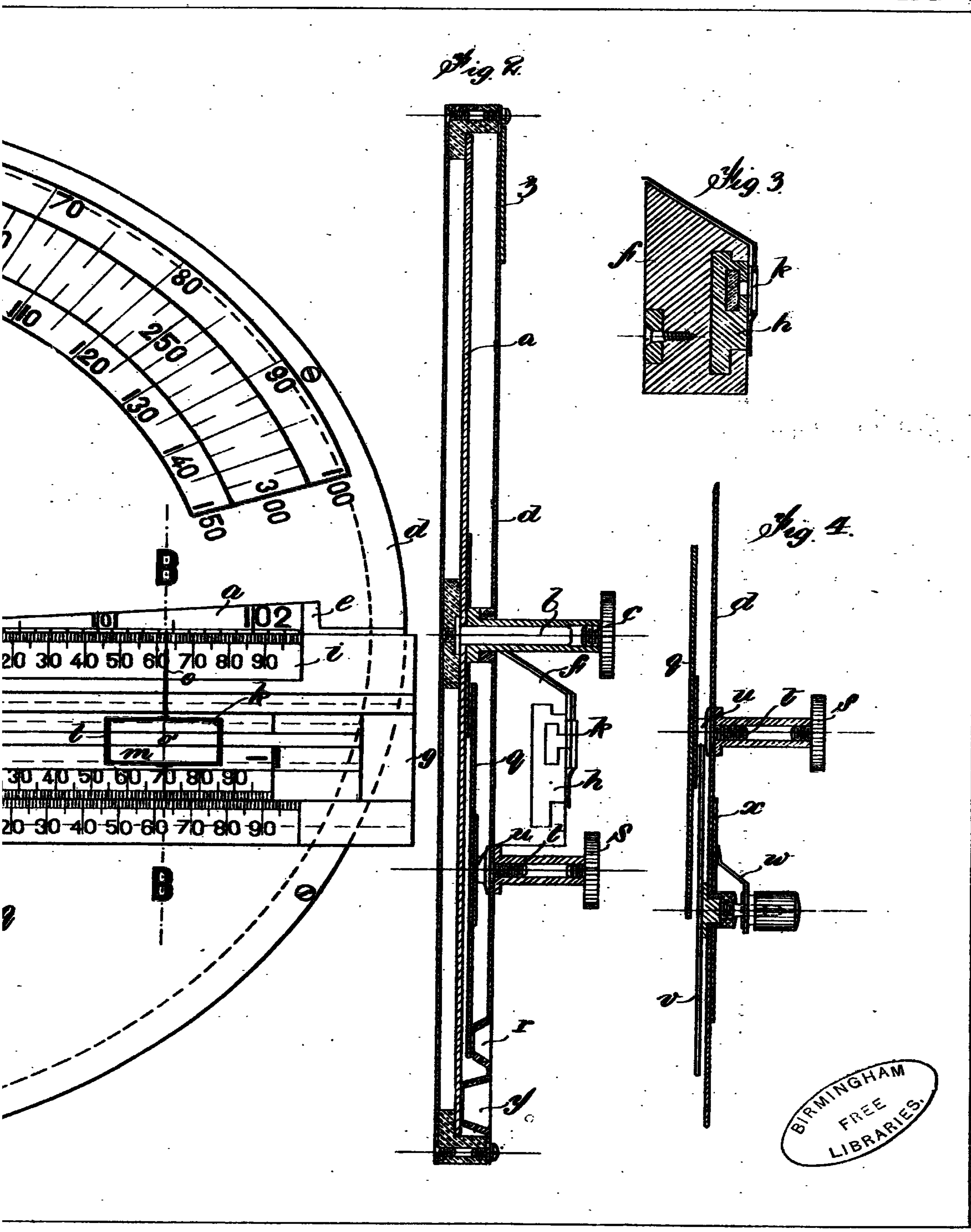


Fig. 1.
A
50
40
60
30
150
100
20
200
70
80
60
90
50
100
50
40
10
30
20
10
b
c
f
h
+2
+1
0
10 20
C
0
1
2
3
4
5
6
7
8
9
10
-1
-2
-3
-4
-5
-6
-7
-8
-9
w
x
v
C
r
y
A



Malby & Sons, Photo-Litho.

[This Drawing is a reproduction of the Original on a reduced scale]

Fig. 5.

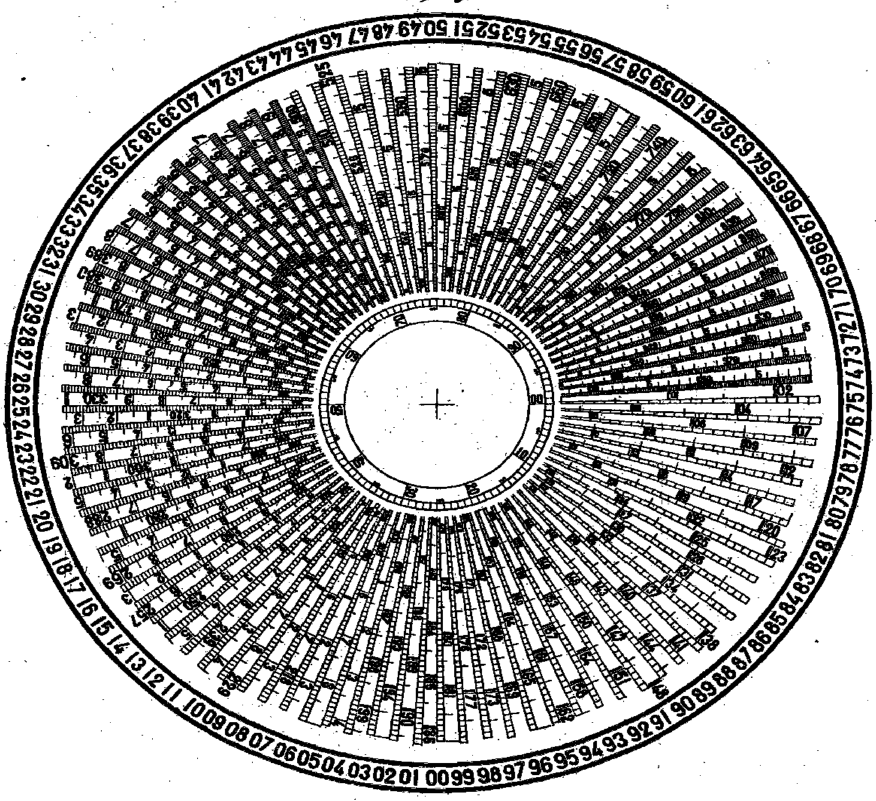
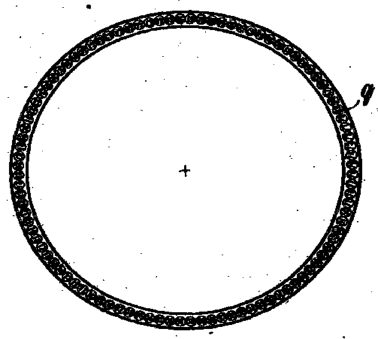
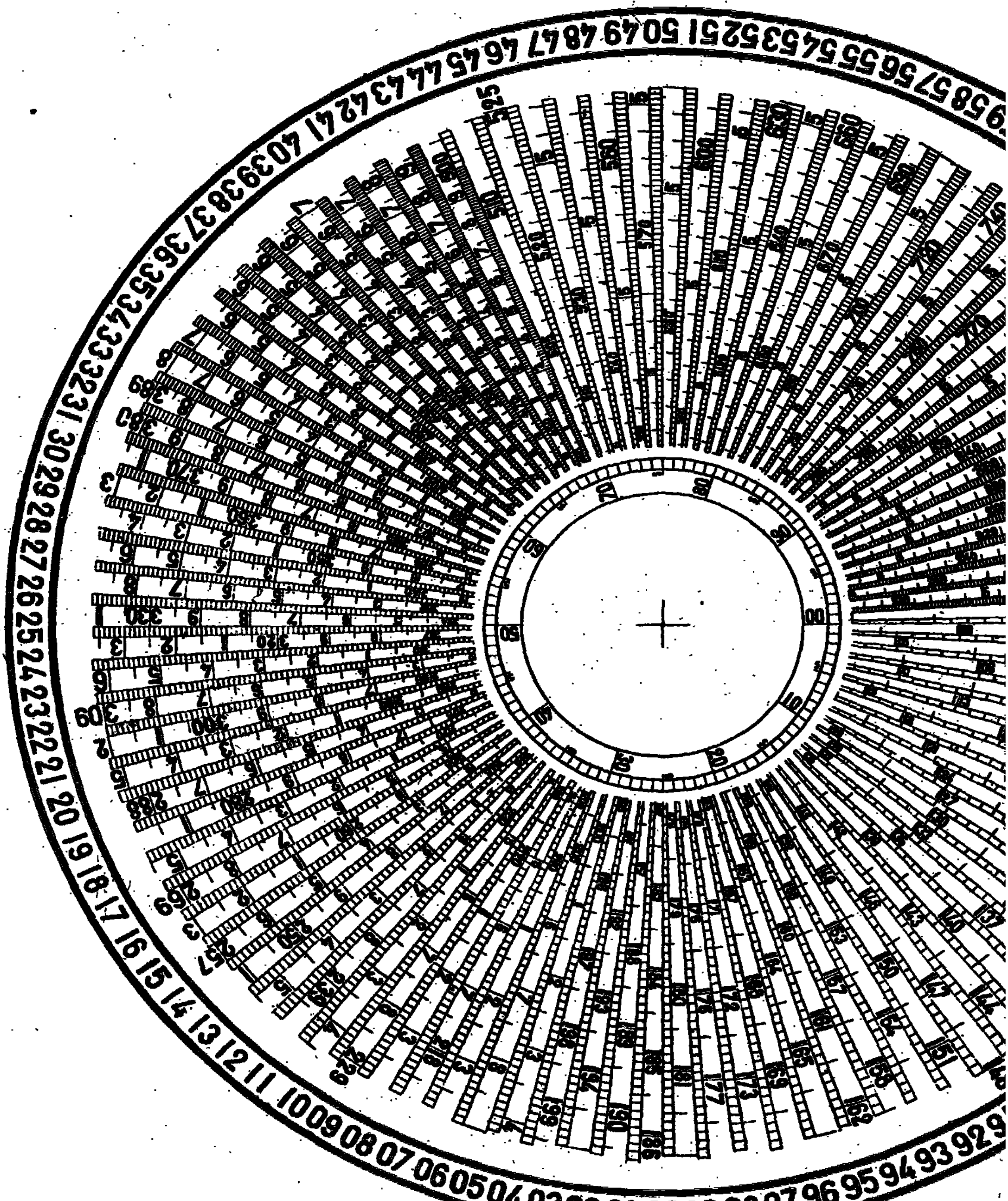


Fig. 6.



Malby & Sons, Photo-Litho.

Fig. 5.



(2 SHEETS)
SHEET 2.

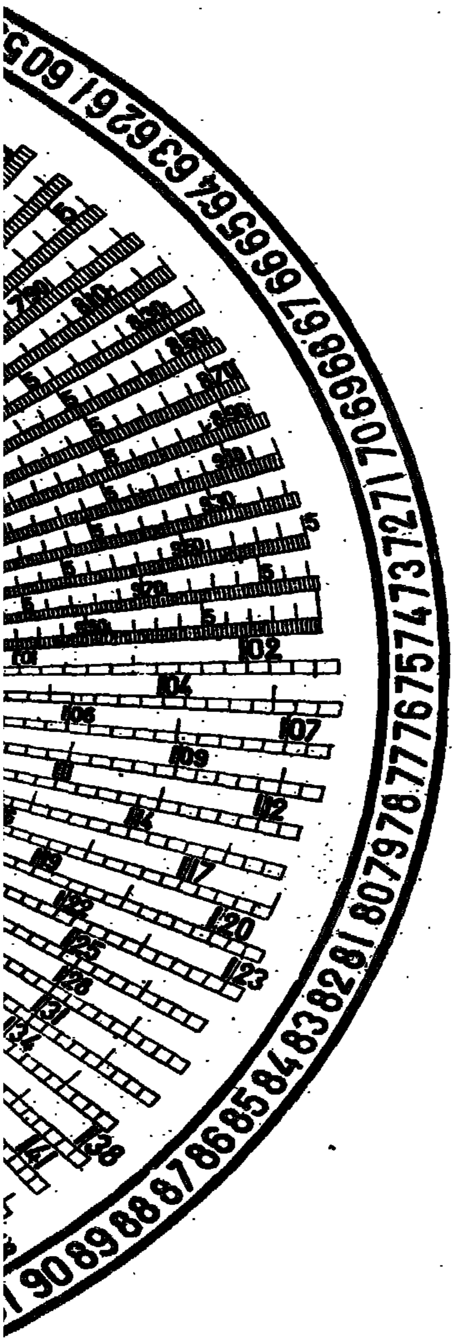
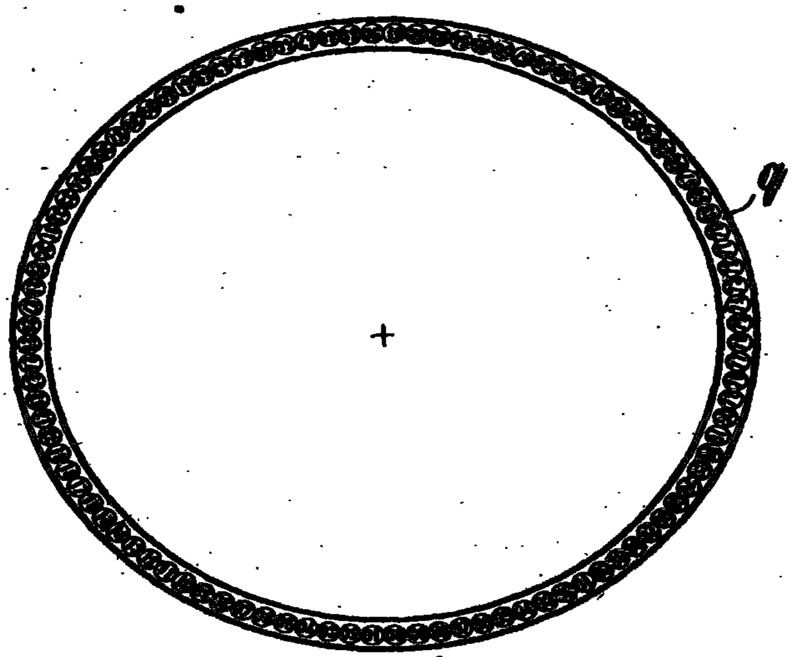


Fig. 6.



Malby & Sons, Photo-Litho.