Circular Slide Rules with Very Long Scales

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Figure 1. The Atlas "Spiral" Slide Rule—Type III.

Introduction

This paper follows a series of reports [1,2,3] that I have prepared on long-scale slide rules—the primary focus of my slide rule collection. Originally, I intended this report to be limited to a comprehensive listing and brief descriptions of circular slide rules with very long scales (VLS), scales with effective scale lengths longer than 4 meters. However, before I got very far in writing, the editor of this fine journal prevailed upon me to report about a previously unknown (to him and to me) VLS circular slide rule that he had recently acquired. In the process of including this slide rule and updating my list of VLS circular slide rules, I became aware of several others that had escaped my attention.

Table 1

Maker	Model	Date First Known	Logs	Type	Disk Diameter cm	Number of Revolutions, Rings or Segments	Actual Scale Length m	Begin Scale Effective Length m	Best Resolution Begin. of Scale	End Scale Effective Length m	Best Resolution End of Scale
Delamain	Mathematical Ring	1630	yes	Concentric Scale	est.10.2	10	≈2.4	1.63	1001	3.00	9998
Sutton	spiral slide rule	1663	no	Spiral Scale	34.1	5	2.80	?	1001	3.98	9999
Sexton	Companion	c1900	yes	Concentric Scale	15.6	10	3.30	?	1001	4.11	9999
Dixon	Spiral Multi-Index	1882	yes	Spiral Scale	29.1	10	4.21	?	1001	7.62	9999
Adams	spiral slide rule	1748	?	Spiral Scale	30.5	10	≈5	?	?	?	?
Montaque	Worman C/10/L.E.A.	1944	yes	Concentric Scale	27.2	10	5.22	2.88	1001	7.87	9999
Lilly	Improved Spiral SR	c1912	yes	Spiral Scale	34.0	10	5.33	?	1001	9.06	9999
Nicholson	spiral slide rule	c1797	yes	Spiral Scale	20.6	10	6.10	1.90	1001	10.63	9999
ARC	Extended Slide Rule	1959	yes	Concentric Scale	25.4	25 semi-circle	6.35	2.51	10002	9.70	99995
Courvoiser	Cercle a Calculs	c1944	yes	Spiral Scale	24.0	20	8.00	4.77	10001	11.78	99995
Ross	Precision Computer	c1914	yes	Spiral Scale	21.1	25	9.14	3.01	10001	15.73	99998
Fearnley	Universal Calculator	1881	yes	Spiral Scale	est.34 x 34	20	≈10	?	?	?	?
Gilson	Atlas Slide Rule	1930s	yes	Spiral Scale	21.1	25	10.67	6.27	10001	15.24	99998
Gilson	Atlas Calculator	1930s	no	Spiral Scale	21.1	30	11.86	6.26	10001	18.76	99998
Gilson	Square Atlas	1920s	no	Spiral Scale	24.8	30	14.00	7.34	10001	22.14	99998
K&E	Mannheim (10-inch)	c1900	yes	Straight Scale	na	1	0.25	**0.25**	1002	0.25	998
K&E	Thacher	1883	yes	Cylindrical	na	40	9.14	9.14	10001	9.14	99995

I have been collecting long-scale slide rules and information on them for more than ten years, so these new additions to my list came as a pleasant surprise. Two of these popped up on eBay. I found the third during an Internet search, a fourth in the depths of my files, and the fifth by pure happenstance while researching a patent. The eBay offerings were a French circular slide rule with an 8m long spiral scale, the *Courvoisier Cercle à Calculs*, and a British circular slide rule with a 5.2m long concentric scale set, the *Montaque Worman C/10/LEA*. Both of these slide rules date to the period immediately after WWII. The Internet find was an Irish circular slide rule with a 5.3m long spiral scale, the *Lilly Improved Spiral Slide Rule*. It was patented in 1912. In my files, I found details in some advertising material for the Sexton *Companion* long-scale concentric-scales slide rule. It dates to the early 1900s, and has a scale length of about 3.3m in ten circular segments. Finally, in a patent search, I found a semi-circular concentric-scales slide rule, the ARC Extended Scale slide rule, which has a scale length of about 6.4m. I will describe these slide rules and others in more detail, but first give some definitions and a brief history of circular slide rules with long scales.

Types of Long-Scale Circular Slide Rules

There are three types of long circular slide rule scales: 1) the basic spiral scale that winds its way continuously towards the outside edge of a disk (an example is the Gilson Atlas Slide Rule); 2) a set of concentric scales on a disk, each scale being a segment of the calculating scale (an example is the Fowler Long Scale Calculator); and 3) a hybrid of the first two types that has a short transition segment from one circular segment to the next to make a continuous scale (an example is the Dempster RotaRule). Most long-scale circular slide rules are of the first two types. The three different scale types are found on two different forms of circular slide rules: 1) one made of a single disk with spiral or concentric scales and two cursors (an example is the Gilson Atlas); and 2) the other comprised of two disks, one smaller disk nested in a larger one. Both disks have matching spirals or sets of concentric scales making up the long scale. Only a single cursor is needed to make the calculation (an example is the Dempster RotaRule).

Effective Scale Length of long-scale Circular Slide Rules

Gilson advertised [4] that the length of the spiral scale on its *Atlas Slide Rule* was 50 feet (15.2m) long. Gilson was actually expressing its effective length because the actual scale length is about 35 feet (10.7m). The *effective scale length* of any discrete section on a long-scale circular slide rule with a spiral scale (or a scale made up of a set of concentric rings) can be determined by comparing that section with a comparable section along an ordinary 25cm-long scale. The effective scale length is the ratio of the section length on the long-scale circular slide rule divided by the length of the comparable section along a C or D scale on the ordinary slide rule times 25cm. This calculation gives the full length of a straight logarithmic scale that would have the same length in that section. Because spiral and concentric scales expand in diameter as they work their way outwards, each new winding or ring is longer than the previous one. This results in effective lengths that are always greater at the high end of their scales than their actual scale lengths.

If we examine the effective length of the spiral scale on the *Atlas Slide Rule* (Figure 1), we will see that the effective length increases logarithmically as we move along the scale from 1 to 10. To facilitate this analysis, we break the logarithmic scale into many discrete sections, and calculate the effective scale length for each section. Near the beginning of the scale the effective length is about 6m, whereas near the end of the scale the effective length is greater than 15m, or about 50 feet as Gilson claimed in his promotional literature. The actual length of the Atlas spiral scale is between these extremes, about 10.7m. Figure 2 shows that the lower third of the scale has an effective length less than the actual length, and the upper two thirds has an effective scale length greater than the actual length.

What does this mean for the precision of spiral scales or concentric scale sets? We can see the effect by comparing the scales on the Ross spiral slide rule (Figure 3) and the Thacher cylindrical slide rule. The actual scale lengths of the Ross and the Thacher slide rules are identical = 9.1m (Table 1). For the purposes of this discussion, we assume that the scale runs one log cycle from 10000 to 100000. At the high end of the scale, the physical distances between the 99000 and 100000 gradation lines are 6.8cm and 4.0cm respectively, the spacing on the Ross being more than 70% greater than that on the Thacher. This means that the Ross has an effective scale length at its high end of about 15.7m versus the 9.1m effective scale length for the Thacher.

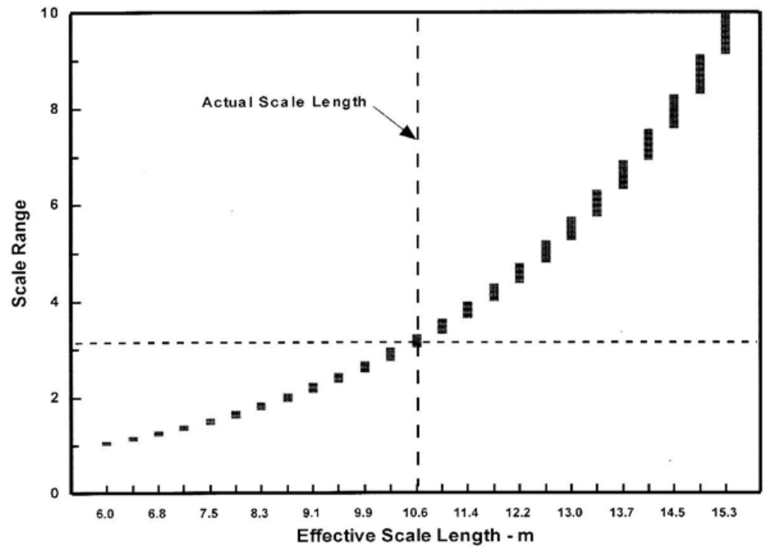

Figure 2. Effective length for the Atlas Slide Rule along the length of its scale.

Figure 3. The Ross "Spiral" Precision Computer. Note smaller attached linear slide rule.

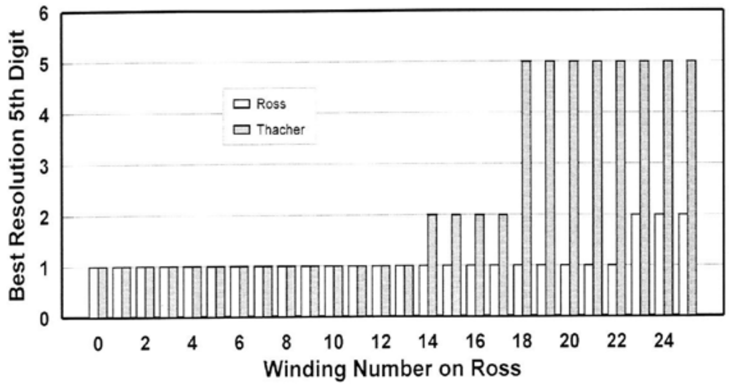

Figure 4. Comparison of the precision of Ross and Thacher scales.

A comparison of the relative precisions of the Ross and Thacher slide rules is illustrated in Figure 4. One can see that the Ross can be read to fully five digits for most of its scale; i.e., one can read five digits to the nearest units digit of 1 for most of the scale. Only near the end of the scale does the resolution fall to the nearest units digit of 2; i.e., one can resolve 99990, 99992, 99994, 99996 and 99998 or 100000, but not 99991, 99993, 99995, 99997 or 99999. For the Thacher, the precision begins to diminish near the middle of the scale and at the end of the scale the precision is only to the nearest units digit of 5; i.e., one can read precisely 99990, 99995 or 100000, but not 99991, 99992, 99993, 99994, 99996, 99997, 99998 or 99999. For all of its complexity and large size, readings on the Thacher cylindrical slide rule are less precise than on the Ross spiral slide rule. This analysis demonstrates the beauty (and advantage) of the spiral and expanded ring scales.

Early History of the Development of the Long Scale Circular Slide Rule

Before describing some important VLS circular slide rules, it is interesting to review their history. It all starts with Napier's invention of the logarithm table and Gunter's laying out the table in the form of a logarithmic number scale. The critical next step was made by William Oughtred in the early 1620s, when he put two rules with matching *Gunter* scales side by side to form a primitive slide rule. Oughtred soon after invented the circular form of the slide rule, and instructed his math students on how to use them. We know this history and much of the history of slide rules up to the early 1900s from Cajori [5,6].

Who Invented the Long-Scale Circular Slide Rule?

Who should get the credit for the early development of the long-scale circular slide rule has been controversial. William Oughtred is credited by Cajori [6] for developing both the linear and circular slide rule forms during the 1620s. While Oughtred did not publish his work on slide rules, William Forster translated his notes and published a description of his instruments and how to use them [7] in 1632. A controversy arose when one of Oughtred's students, Richard Delamain, published an essay [8] a year earlier in which he claimed to have developed several forms of the slide rule, including the circular style that Oughtred invented during the 1620s. Cajori [6] concludes that Delamain stole this invention and many other ideas from Oughtred.

Oughtred called his circular slide rule the *Circles of Proportion.* His design included a number of concentric circular scales on a brass disk—each scale being for a different mathematical operation. According to Cajori [6] Oughtred gives credit for the invention of the spiral scale to Thomas Brown, an instrument designer and maker of his time. The spiral scale was a leap forward in slide rule design, because it allowed very long scales on an ordinary circular slide rule disk. It is remarkable that this scale design came so early.

However, regarding the design of the form of the long-scale slide rule made up of a set of concentric scales, it appears that Delamain should get the credit. There is no evidence that Oughtred considered this form, or that Brown made long-scale slide rules with the concentric scale format. It seems, then, that while Oughtred planted the seed for long-scale circular slide rules with his invention of the circular slide rule, Brown and Delamain deserve the credit for taking the next step.

Spiral and Expanding Ring Slide Rules in the 17th to the 19th Centuries

Thomas Brown Spiral Slide Rules

As discussed earlier, the first spiral slide rule was made by Thomas Brown in 1631. Brown was an independent instrument maker who made slide rules and other instruments for William Oughtred and others. However, not much is known about these early spiral slide rules except that they were made with "5, 10, and 20 turns", and that they were of the single disk type with "flat compases" for cursors [6]. This design was a natural extension of the Gunter rule design where compasses (dividers) were used to make calculations along straight logarithmic scales.

Richard Delamain's Mirifica Logarithmoru' Projectio Circularis

Richard Delamain, in his series of essays, entitled *Grammelogia, or the Mathematical Ring,* published drawings of concentric scale circular slide rules. The first edition [8] was printed in 1630, the year before Thomas Brown invented the spiral slide rule. I do not have access to the drawings in the 1630 publication, but Cajori [6] showed two different drawings from what he called *Grammelogia IV* [9], which was printed in 1632 or 1633 (the date is uncertain). The second of the two plates is dated 1630, so both may have been printed in *Grammelogia I,* which was published in 1630. Both drawings show scales that are uniquely different from Oughtred's and Brown's designs.

Figure 5. Richard Delamain's Mirifica Logarithmoru' Projectio Circularis double ring circular slide rule with concentric scales.

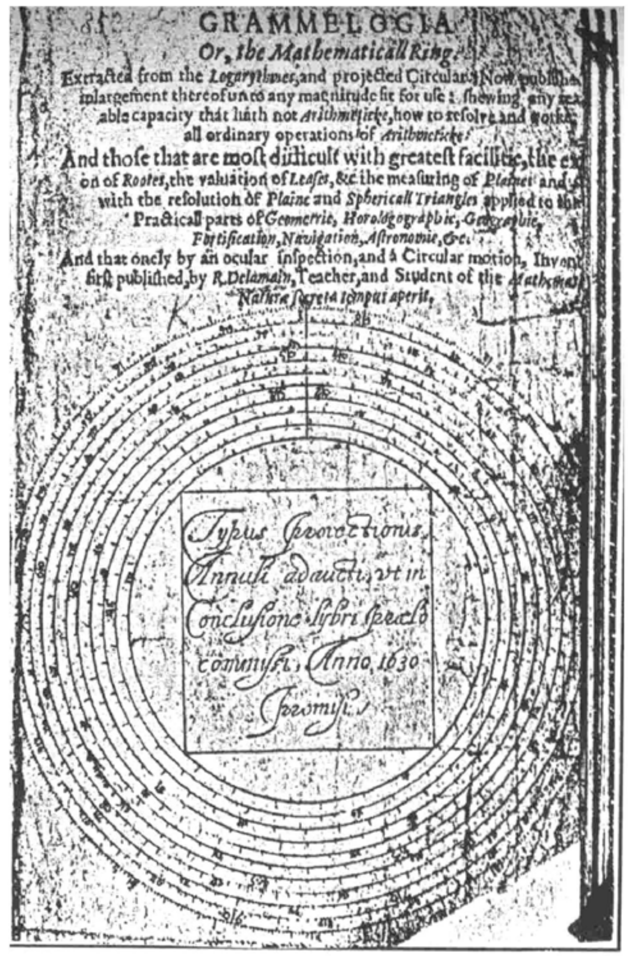

Figure 6. Richard Delamain's Mirifica Logarithmoru' Projectio Circularis single disk circular slide rule with concentric scales.

The scales on the first illustration (Figure 5) are not all on a single fixed disk, but are laid out on two coplanar disks. Each disk has a concentric scale set made up of five circles. The two disks are nested; the smaller diameter inner disk rotates relative to the larger disk to make calculations much as the Dempster RotaRule that we know of from the 20^{th} century. It appears that the scale works its way inward on the inner disk and outward on the outer disk—so that the adjoining pair of scales match each other. Cajori [6] reported that the outside diameter of Delamain's Mathematical Ring was 4 inches (10cm), and that the disk was made of brass. I have calculated that the average length of the inner and outer scales would have been about 1.25-m.

The second of Delamain's drawings (Figure 6) shows a circular slide rule made up of a set of concentric scales on a single disk, the scale expanding outward in ten segments. This design of Delamain's takes advantage of the increasing precision obtained when the circular scale sections increase in length as they work their way outwards on the disk. Assuming that the disk diameter was also 4 inches (10.2-cm), the scale length would have been about 2.4m. This is the earliest long-scale circular slide rule for which we have sufficient scale design and length details. It does not quite meet my criteria for a VLS circular slide rule because the best resolution near the end of the scale (on the outermost ring) is a bit less than four digits (9998, not 9999). Its effective scale length near the end of the scale is about 3m, less than the 4m estimated break point for VLS circular slide rules. If the outer ring were about 14cm in diameter, then the effective scale length would have been about 4m.

The outer edge of both of the Mathematical Ring slide rules also appear to have circular scales with linear gradations, perhaps for determining the mantissas of logarithms. We will see later how this is done. It should be noted that while not shown in Figures 5 and 6, Delamain's two-disk design would have required a single cursor, and the single-disk design two cursors.

Until recently no examples of Delamain's circular slide rules were known. However, Peter Hopp [10] reported a communication from Francis Wells in France that the book by Wedgewood [11] *The Trial of Charles I* tells of King Charles making a gift of *"a large double ring of silver with figures engraved on it, which could be used as a sundial and a slide-rule "resolving many questions in arithmetic". It was the invention of Richard Delamaine, who long ago, had taught mathematics to King Charles".* So it appears that at least one example of Delamain's *double ring* slide rule was made. Given Peter Hopp's usual persistence in tracking down the rare and unusual slide rule, we soon should know much more about Delamain's long-scale circular slide rule.

William Milburne Spiral Slide Rule
Hopp [12] and Cajori [6] both reported that William Milburne designed a spiral slide rule in 1650. However no details are available. Hopp thought that Thomas Brown may have made spiral slide rules designed by Milburne. In any case little is known of Milburne or Thomas Brown's contributions.

Remarkably, the Science Museum in London has on display two early spiral slide rules dating to the 1660s, including an original [13] by John Brown (a son of Thomas Brown) that dates to about 1660. The other is a spiral slide rule [14] signed Henr Sutton fecit, and dated 1663. The John Brown spiral has a calculating scale length of about 2.1m in five revolutions, and the Sutton has a scale length of about 2.8-m, also in five revolutions. Both also have sine and tangent spiral scales.

John Brown Spiral Slide Rule
The John Brown spiral scale is made on a wooden disk, about 15cm diameter. This disk has three spiral scales: 1) the innermost one a 7-revolution tangent scale; 2) the outermost scale being a 5-revolution number scale; and 3) a 5-revolution sine scale between these two scales. The scales all wind their way outwards on the disk. A pair of brass indicator arms facilitates the calculations. The length of the spiral number scale is about 2.1m. Its effective length is about 2.4m. With care, the John Brown spiral slide rule can be read to nearly four digits at both ends of the scale.

Hopp [12] reported that John Brown coined the name spiral "slide rule" for his circular slide rule, even though in a strict sense, there were no sliding parts, and the circular slide rules were not linear in layout as the word *rule* would infer. This name, however, has stuck, and we still use it to refer to calculating disks with logarithmic scales. The only known example of a John Brown spiral slide rule is the one in the Science Museum in London. I showed a photograph of it in an earlier paper [3].

Henr Sutton Spiral Slide Rule
The Sutton spiral slide rule is laid out on a brass disk about 34cm diameter, more than double the size of the John Brown wooden disk. The scale layout is most curious. It starts near the center of the disk with four windings of a pair of scales for calculating functions of angles. The number calculating scale starts at the beginning of the fifth winding and continues for five additional windings, while the angle scales continue for an additional two revolutions. A pair of brass indicator arms facilitates the calculations. The length of the number scale is about 2.8m. Its effective length is about 4m; long enough to qualify the Sutton as a VLS slide rule. It is the first VLS spiral slide rule that we have details for. The Sutton spiral slide rule can be read with interpolation to about four digits at both ends of the scale. The only known example is in the Science Museum in London. A picture of it was shown on the cover in an earlier issue of this journal [3].

A few new circular slide rules with spiral or concentric scales appeared over the next 200 years, almost always as independent new designs, each designer having little apparent knowledge of earlier work.

Facini Nu. De Logaritmi
In 1717 in Italy, Bernardus Facini designed a spiral slide rule that had a scale length of about 1.2m (1.9m effective length) in four revolutions on a brass disk. Two brass cursors facilitate the calculations. Interpolation of readings is aided by the inclusion of vernier-like markings that run on a band that winds just outside the spiral scale. The reverse has a four revolution spiral scale for addition and subtraction. The only known example of Facini's slide rule is in the Adler Planetarium & Astronomy Museum in Chicago. A photo of the front of this disk appeared on the cover of the Journal of the Oughtred Society [15].

Adams Spiral Slide Rule
In 1748, George Adams made a spiral slide rule on a 25cm diameter brass plate with 10 windings of the scale [5]. No examples or other details are known.

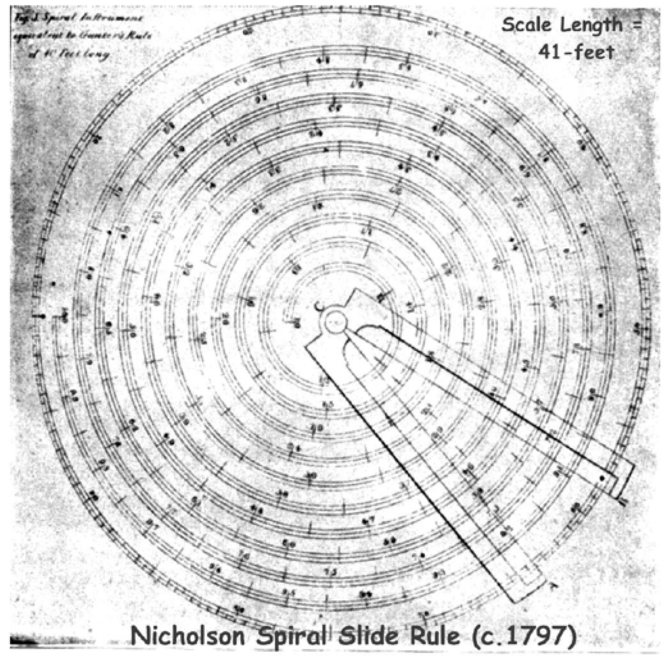

Figure 7. Nicholson's spiral slide rule.

Nicholson Spiral Slide Rules
The next designer of long-scale circular slide rules that we know of is William Nicholson. In 1797 he described a spiral slide rule with ten windings and a scale length "equivalent to a Gunter rule 40 feet long" (or a single-decade log scale 20 feet [6.1m] long). Its effective length is about 10.6m. One can fully resolve four digits at both ends of the scale. The Nicholson (Figure 7) slide rule also has a circular scale with linear gradations running just outside the last winding of the spiral scale. This is called a log scale, for it is used to determine the logarithms of numbers. The log scale runs from 0 to 100, and is used in concert with the spiral scale to find logarithms of numbers. Four-place logarithms can be found. The first digit of the mantissa is the winding number of the spiral scale, the winding numbers starting with 0 and running to 9. The next three digits are found on the outer ring. It is uncertain if this is an original idea of Nicholson's, as Delamain's long-scale concentric scale slide rule appears also to have a circular scale with linear gradations running near the outside edge of the disk. Like many of the earlier spiral slide rules, no examples of the Nicholson slide rules are known. Cajori [5] however showed a detailed line drawing (Figure 7) done by Nicholson.

There seems to have been a long break in the evolution of the spiral slide rule after Nicholson's contributions in the late 1700s until 1881 and 1882, when Fearnley and Dixon designed their spiral slide rules.

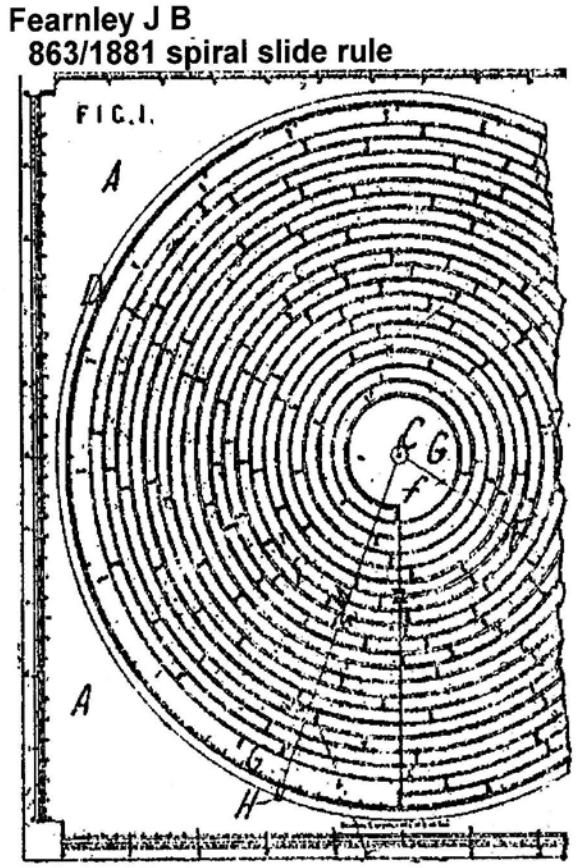

Figure 8. Fearnley's Universal Calculator from patent.

Fearnley's Universal Calculator

J.B. Fearnley patented [16] his spiral slide rule in 1881. Pickworth [17] cited the Fearnley's Universal Calculator as an example of a spiral form of long-scale slide rule, so we know that some were made, but do not know of any existing examples to provide details. The patent shows (Figure 8) a spiral slide rule with 20 windings laid out in a rectangular plate. No dimensions are given so there is an uncertainty about the length of the spiral and its long-scale characteristics. If the plate dimensions were 34 x 34cm like the diameter of the Sutton spiral slide rule, then the scale length would have been about 10m. The patent drawing also shows a linearly divided ring running outside of the spiral scale, but is too small to gather any details about effective length and scale resolution. The patent describes how calculations are made with the spiral scale and how mantissas for logarithms are determined using both the spiral and the outer ring scales.

Dixon Combined Circular, Spiral, Multi-Index Slide Rule and Four-Figure Logarithmic Decimal Scale Table

An original framed version of the Dixon spiral slide is in the Science Museum [18] in London along with the early Brown and Sutton spiral slide rules. A picture of the Dixon spiral slide rule was shown in my previous *JOS* article [3] on long-scale slide rules. The Dixon spiral has a scale length of about 4.2m (and an effective scale length of about 7.6m), the scales and markings being printed on paper mounted on a board in a frame. In addition to the spiral scale, the Dixon slide rule has two circular scales that run outside the spiral; one a single-cycle log scale labeled *numbers* running from 1 to 10, and the other, labeled *common logarithms*, linearly divided in 500 parts from 1 to 10. The *number* scale is used to multiply and divide to a precision of about three digits, while operations with the *spiral scale* provide results to about four digits. The *common logarithm* scale is used to help find the logarithms of numbers to four places. Three brass cursors facilitate operations. The brass cursors are marked along their lengths from 0 to 9 to enable the finding of logarithms of numbers. Each of the numbers lines up with a winding of the spiral scale. Like the Nicholson design of 1797 and the Fearnley design of 1881, these markings are used in concert with the logarithm scale to determine the mantissa of a logarithm, the first digit obtained from the cursor markings, and the next three digits from the *common logarithm* scale. A photo of the Dixon slide rule was shown in an earlier issue of this *Journal* [3].

Dixon designed several other slide rules including reportedly a VLS circular slide rule with the concentric scale format. Robinson [19] gave a brief description of a Dixon concentric scale slide rule after viewing one at the Inventions Exhibition in London in 1885. Robinson described this as a special rule with the scale extending over a set of ten concentric scales—the inner circle having an equivalent length of 30 feet (4.57m) and the outermost circle having an equivalent length of 60 feet (9.14m). He gives no other details, but the size of this instrument must have been about the same as the size of the Dixon spiral slide rule. It is curious that Robinson did not report seeing Dixon's spiral slide rule as it had clearly been designed by the time of the London Exhibition of 1885. Could it be that Robinson saw Dixon's spiral scale and just misunderstood it as a long-scale slide rule made up of concentric scales?

Very-Long-Scale Circular Slide Rules in the 20th Century

The first half of the 1900s was an explosive period of slide rule development, for all kinds of slide rules. That includes circular slide rules with spiral and concentric scales. We know a lot about the Fowler long-scale slide rules and many other "pocket" circular slide rules with relatively long spiral scales or concentric scale sets. I reported on them in *JOS* [1] and *JOS* [3]. More interesting to me are the *very-long-scale* spiral or concentric scale slide rules—circular slide rules with effective scale lengths 4m (13 feet) or longer that can give results to at least four significant digits.

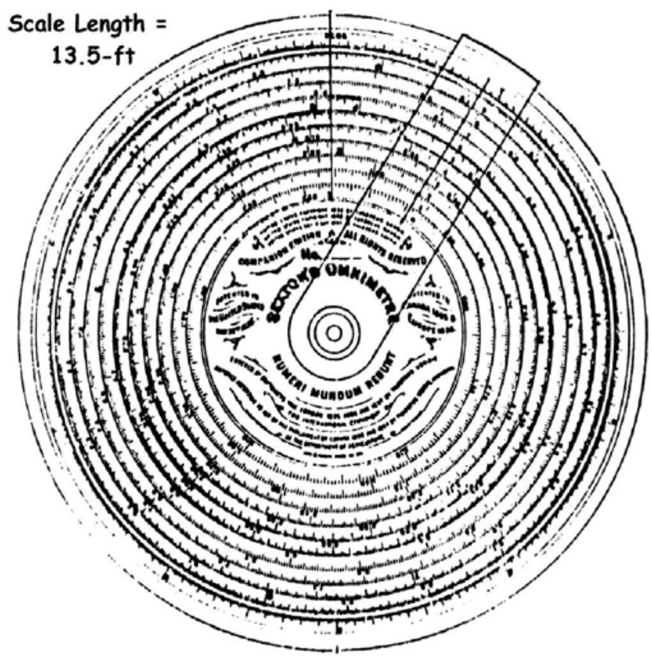

Figure 9. Sexton Companion concentric scale slide rule.

Sexton Companion Concentric Scale Slide Rule

Perhaps the first VLS circular slide rule of the 20th century was the Sexton #6 *Companion* circular slide rule (Figure 9) with a scale made of 20 concentric scales having an effective length of about 4.1m. This slide rule was intended to be a companion to a regular Sexton *Omnimetre* circular slide rule—more precise calculations (to about four digits) being made on the *Companion* model once the approximate result was known from the *Omnimetre* model. Like the Nicholson and the Delamain slide rules, the Sexton has a linear scale near the outer edge of the disk for determining logarithms of numbers. Ring numbers from 0 to 9 are printed along the transparent cursors for determining the first digit of the mantissa. I know of this slide rule only from advertisements [20] for the *Omnimetre* models. It was made from a heavy card stock and had a diameter of about 15.5cm.

The Sexton *Omnimetre* model #3 also has an concentric scale set, made up of five nested rings, but it cannot be used as a long scale for multiplication and division without an additional cursor. The directions for using the Sexton *Omnimetre* only illustrate examples of using this scale for finding powers of 5 and 5th roots. While the #3 *Omnimetre* has only one cursor, a second cursor could be added to allow the use of the segmented scale. In this case the scale length would be 95cm versus 48cm for the ordinary single ring scale on the *Omnimetre*, and the effective scale length would be over 1.1m. This would allow three significant digits to be read on both ends of the five-section scale, while three significant digits can be read only on the lower end of the single section scale.

Figure 10.

Lilly's Improved Spiral Slide Rule

Lilly's Improved spiral slide rule (Figure 10) was unknown to me until recently. I found reference to it while making a last-minute search on the Internet for spiral slide rules. Dr. Charles Mollan, editor of *The Irish Scientist*, kindly provided details to me [21]. Dr. Walter Lilly designed his slide rule in his role as an instructor of engineering at Trinity College in Dublin, Ireland, and obtained a British patent [22] for his design in 1912. Lilly laid out his spiral scale on a paper surface glued to a wooden disk about 34cm in diameter. Like the Nicholson and Dixon models, the Lilly disk has a spiral scale with ten windings and a circular scale with linear divisions outside the last winding of the spiral scale. The length of spiral scale is about 5.33m (effective length $\approx$ 9.1m), and as with the Nicholson, Dixon, and Sexton models, the logarithms of numbers can be obtained, in addition to products and quotients. Instructions [23] are printed on the reverse side of the disk. Dr. Mollan knows of two examples of the Lilly slide rule, one in the collection of the Civil Engineering Department, Trinity College, Dublin, Ireland, and the other is in his collection and currently in a Museum display at St. Patrick's College in County Kildare.

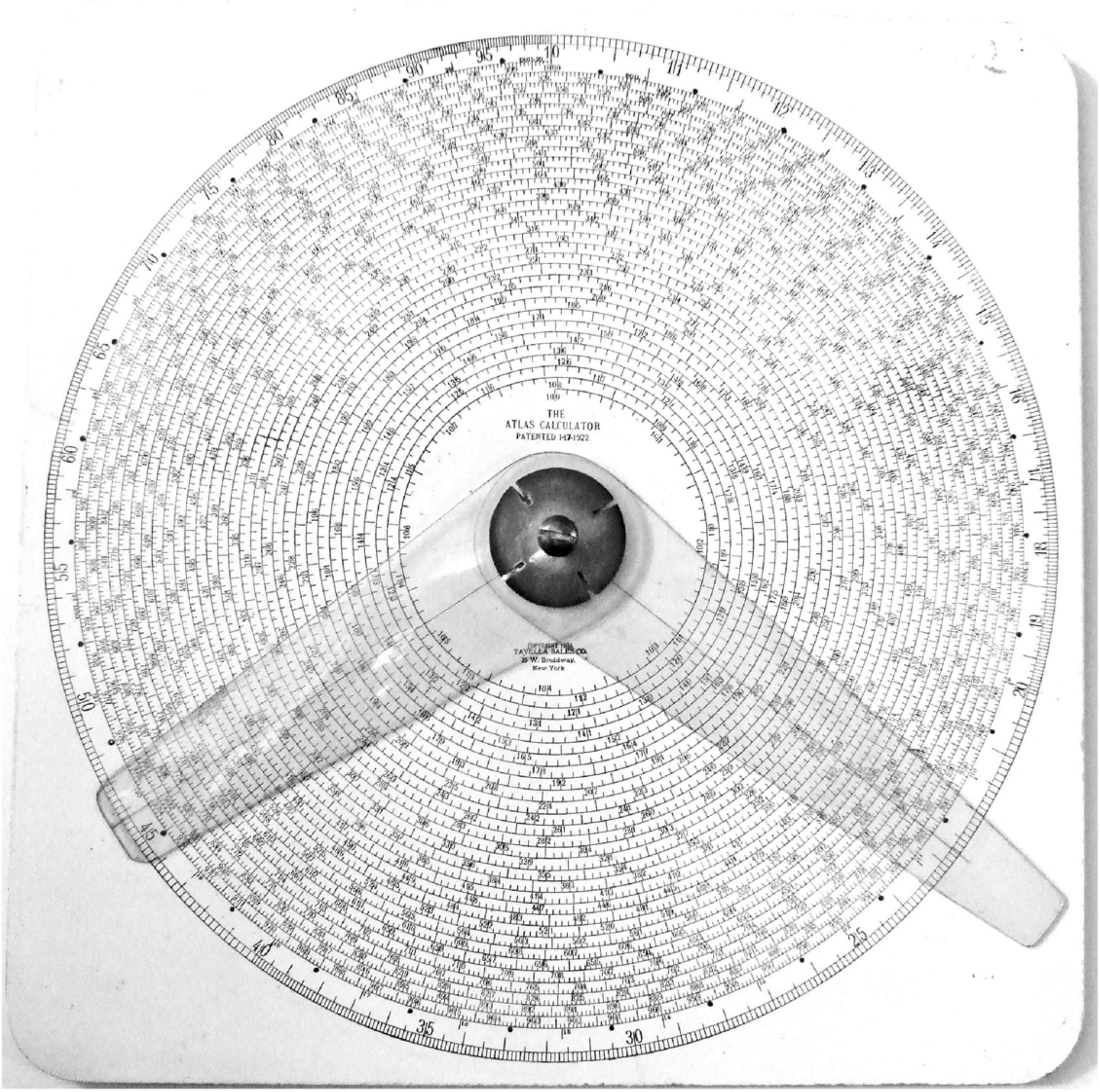

Figure 11. Gilson "Square" Atlas spiral slide rule—Type I.

Gilson Atlas Spiral Slide Rules

The Gilson Atlas spiral slide rules were probably the most widely sold (and are currently the most readily collectable) VLS spiral slide rules ever made. I discussed the Atlas models in an earlier *JOS* paper [24], but will go into more detail here.

The Atlas was made in three sizes. The spiral on the Type I *Square* Atlas (Figure 11) was laid out in 30 windings on a square metal sheet. Its scale length was about 14m (effective length $\approx$ 22.1m), the longest spiral scale slide rule known to me. It was first made in the early 1920s.

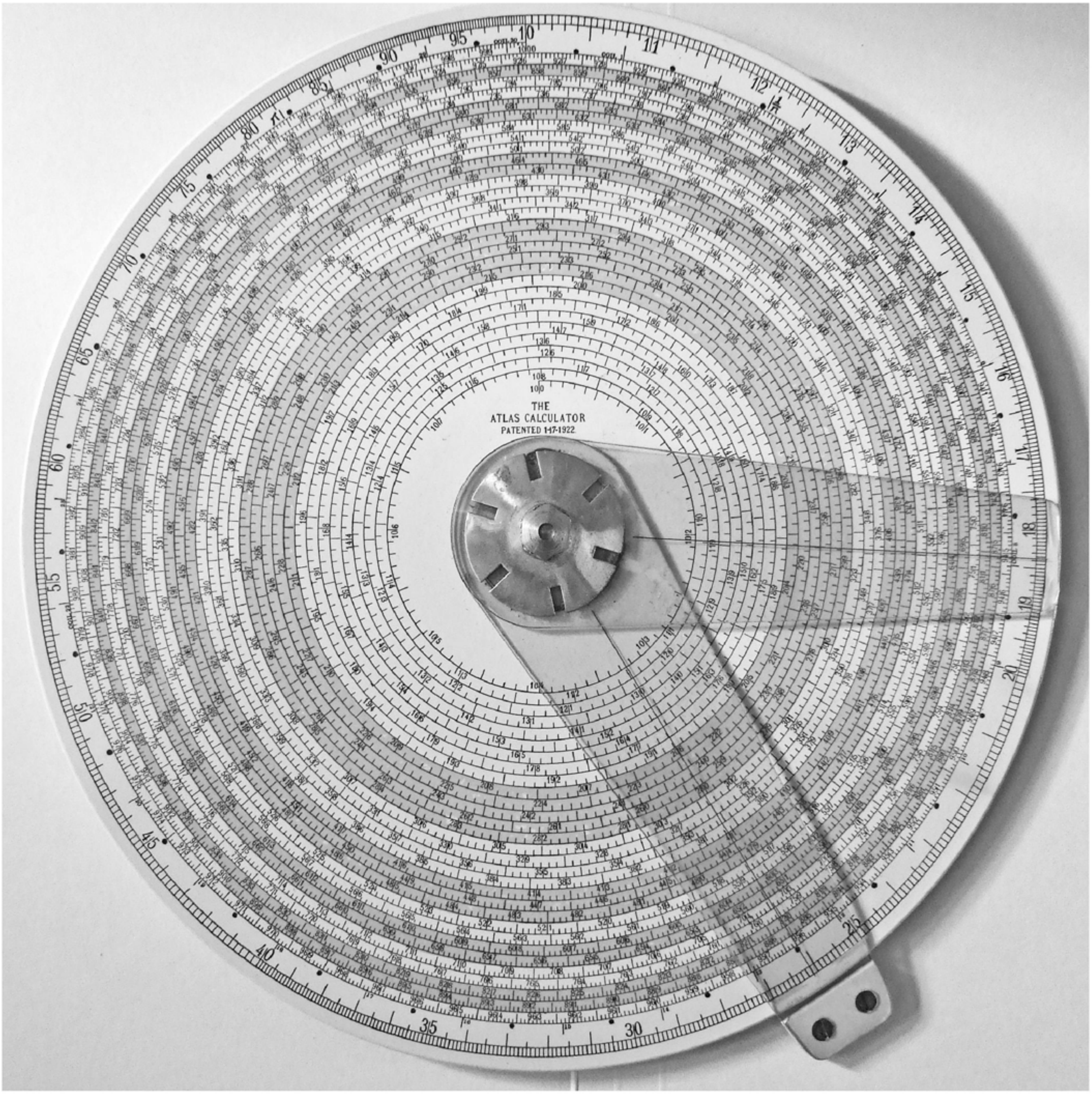

Figure 12. Gilson Atlas Calculator—Type II—spiral slide rule.

Apparently because the Type I was a bit large and awkward to use, Gilson introduced a new model in the late 1920s with the identical scale—but on a smaller 8-inch-diameter disk (Figure 12). He named it *The Atlas Calculator.* I call it the Type II Atlas spiral slide rule. The scale length was about 11.9m (effective length $\approx$ 18.8m).

Gilson got the disk size right for this model, but, maybe because it was a bit difficult to read a spiral scale with such narrowly spaced windings, he brought out a new (Type III) model (Figure 1) in the early 1930s on the same size disk, but with a 25 revolution spiral scale (instead of 30 windings) having a scale length of about 10.7m (effective length $\approx$ 15.2m). He named this one the *Atlas Slide Rule.* This model was available with both plain and highlighted scales, the highlighted scales facilitating the finding of the correct winding. As with the Sexton model #6, the approximate result of a calculation is first found on a simple slide rule scale. However, in the case of the Gilson Atlas, the simpler calculating scale is found at the outside edge of the disk, not on a separate slide rule. The *square* Atlas scale can be read to fully five digits at both ends, while the other two Atlas models both can be read to five digits at the low end of the scale and to nearly five digits at the high end.

On the Type III Atlas Slide Rule, the correct winding (or coil) of the spiral scale can also be determined from a *Coils Scale* located near the center of the disk [25]. The rule for using the *Coils Scale* is always to work the cursors in the clockwise direction. For multiplication, one adds the *Coils*

Scale numbers for each of the numbers in the operation to get the coil number for the product. If the sum of the *Coils Scale* numbers exceeds 25, then 25 is subtracted from it to get the coil number for the solution.

The Type III Gilson Atlas Slide Rule model also has scales for finding the logarithms of numbers to five places. A *C-Log* scale located near the center of the disk is used to determine the first two digits of the mantissa and an S-Log scale located just outside the last winding of the spiral is used to find the next three digits.

Gilson Atlas Slide Rules are often found in collections and for sale. The *Square Atlas* and *Atlas Calculator* are much less common. Even more scarce are versions of the Atlas with Binary scales on the reverse side (rather than the more common set of trig scales).

Ross Precision Computer

The only other VLS spiral slide rule known to me until recently was the *Ross Precision Computer.* The first mention of the Ross Precision Computer is in a *Scientific American* article [26] in 1916. It was designed by Louis Ross, a civil engineer, in San Francisco, California, in about 1915. The Ross spiral scale is laid out on a disk (Figure 3) about the same size as the later Gilson Atlas models. It has a scale with 25 windings and a scale length of about 9.14m (effective length $\approx$ 15.7m). The Ross spiral scale can be read to nearly five digits at both ends. It has a unique feature for determining the approximate value before using the spiral scale, a small rectilinear slide rule that pivots on the axis of the disk. One makes the approximate calculation on the small slide rule first and then on the spiral scale. An arrow on the slide points to the winding on the spiral scale for a more precise result.

The Ross also has provisions for determining the logarithm [27]. The main or *Base* cursor has a *Quadrant* scale graduated in steps of 4 from 0 to 100 from the most inside winding to the most outside winding of the spiral scale. The *Log* scale running near the rim of the disk has a scale divided in 1000 parts. The first two digits of the mantissa are obtained from the *Quadrant* scale and the next three digits are obtained from the Log scale. Like the Gilson Atlas Slide Rule model, logarithms can be determined to five places.

The example of the *Ross Precision Computer* in my collection is made of stainless steel with the scales etched in to the raw metal surface. I also know of one made of brass. Most other examples that I have seen were made of a zinc alloy metal with enameled scale surfaces. These scale surfaces are very susceptible to chipping and flaking when the zinc surface under the paint oxidizes. All of the painted examples of the Ross that I have seen have suffered some surface damage due to oxidation. The Ross saw limited sales from 1915 until about 1924. The painted version of the Ross Precision Computer is a scarce spiral slide rule. Examples made of stainless steel or brass are rare.

Montaque Worman Concentric Scale Slide Rule The Montaque Worman model *C/10/LEA* is a concentric scale type of long-scale circular slide rule with ten nested scales having a total length of about 5.2m (effective length $\approx$ 7.9m). The scales are laid out on a plastic disk (Figure 13) with a diameter of about 10.5 inches. Little is known about the maker of this slide rule. Its address is given near the outer rim of the disk as Alliance Buildings, 4 Mosley St., Newcastle-On-Tyne, 1. It was probably made soon after WWII, perhaps in the early 1950s. In addition to the set of ten concentric scales for multiplying and dividing numbers, the *C/10/LEA* has an outer scale linearly numbered from 0 to 9 that is used to find logarithms of numbers, and a set of smaller counting disks, mounted on a hub at the center of the calculating disk, that are used to help identify the number of the ring on which to find a result. Two transparent plastic cursors with hairlines facilitate the calculations. Instructions [28] accompany the *C/10/LEA.*

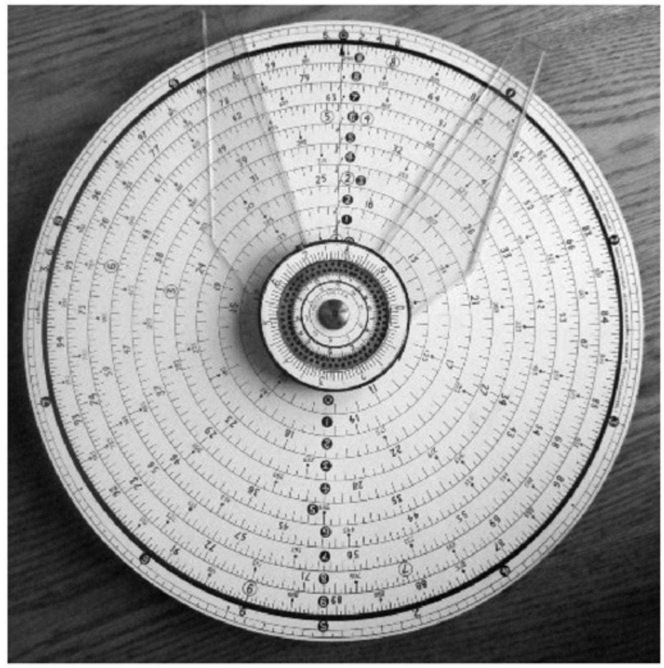

Figure 13. The Montaque Worman Model *"C/10/LEA"* Concentric Scale Slide Rule.

As with all circular slide rules with the scales laid out on a single disk, multiplication is done by first setting the hairline of one cursor on the index (starting point of the scale) and the hairline of the second cursor on the multiplier while holding the first cursor on the index of the scale. The pair of cursors are then turned in unison to position the hairline on the first cursor over the operand number. The result is found under the hairline on the second cursor. The problem of determining which ring the result will be on, when there are ten rings to choose from, can either be done by mentally determining the approximate result or by systematically using the ring numbers. This is accomplished by adding the ring numbers that the multiplier and operand are found on. The procedure is reversed for division.

The center hub rings on the Montaque Worman slide rule are used to assist more systematically with determining on which ring to find a result. The center hub not only facilitates determining on which ring the number is, but also it helps (in concert with a linear scale on an outer ring) to find the relative position. The outer ring is also used to find logarithms of numbers.

The Montaque Worman *C/10/LEA* can be read to a precision of four digits at both ends of the scale. The only known example of the Montaque Worman slide rule is in a private collection [29].

Courvoiser Cercle à Calculs

The Courvoisier *Cercle à Calculs* is a spiral slide rule with

20 windings having a total calculating scale length of 8m (effective length $\approx$ 11.8m). The scales are laid out on a metal disk (Figure 14, inside back cover) about 8.5 inches in diameter. The scale can be read to five digits on the low end and a bit better than four digits at both ends. Little is known about the maker of this slide rule except what was found on a copy of a patent. A French patent [30] was awarded to Monsieur Maurice-Édouard Courvoisier, a resident of Seine, France in 1946. The patent claims that it is a modification of patents awarded in 1844 and 1902.

The *Cercle à Calculs* has two celluloid cursors with hairlines. One cursor has a unique metal fixture with scales fastened to it. I will refer to it as the complex cursor. The spiral scale starts at 100 near the center of the disk and winds its way towards the outer edge to 1000. Outside the last winding of the spiral scale, there is a linear scale labeled from 0 to 50 and 50 to 100. And near to the outer edge there is another scale made up of ten closely spaced rings divided into sawtooth-like scales.

Calculations are made in a way similar to the method used on the *Montaque Worman*, using the hairlines on the two cursors. However, the *winding* numbers are not labeled on the Courvoiser disk as the ring numbers are on the Montaque Worman disk. One finds the appropriate winding using the metal transverse fixture attached to the complex cursor. This fixture has logarithmic scales on its surface, much like the upper and lower stators of a conventional slide rule, in this case both scales having two log-cycles running in an outward direction. The cursor window and hairline are visible between these scales, as are the spiral scale windings. In the starting position, the gradation marks of the first log-cycle on the fixture match up with the corresponding winding on the disk. When making a calculation, one first makes an approximate calculation using the transverse fixture, and then makes the more precise calculation with the spiral scale. The result is then read on the spiral winding under the hairline of the complex cursor.

Logarithms of numbers can also be found with the Courvoisier *Cercle à Calculs* in a method similar to the one used on the Montaque Worman. However, the scale windings are not numbered, and there are 20 windings instead of 10 rings. The sliding fixture is used to determine the first digit of the mantissa, and the linear ring or log scale near the outside of the disk is used to find the next three digits. Because there are 20 windings, the outer scale runs from 0 to 50 for one revolution (ten windings of the spiral scale) and from 50 to 100 for a second revolution for the last ten windings of the spiral scale. To find a logarithm of a number, one first needs to remove the sliding part of the transverse fixture, turn it over to reveal a linear scale, and reinsert it. The linear scale on the fixture gives the first digit of the mantissa, and the log scale gives the next four digits. Like the Montaque Worman, I know of only one example; in this case, the one in my collection.

ARC Extended Slide Rule

The ARC *Extended Slide Rule* is a concentric-scale, semi-circle slide rule (Figure 15, outside back cover). The scales are printed on an enameled aluminum surface. Twenty-five semi-circular concentric scales form the calculating scale. The scales cover a bit more than a semi-circle of 180°. I measured about 197°. This is a unique form of the circular slide rule designed and patented by Andrew Frankenfield in 1961 [31]. Two cursors allow for calculations. The advantage of this semi-circular format is that it has a very long scale (6.35m) in a pocket sized slide rule. The effective length is 9.7m. Solutions can be worked to better than four significant digits on the ARC. The disadvantage over other circular formats is that the scale segments are not continuous. One must at times reverse the direction of the reference index to stay on scale, much as one must do with the slide on an ordinary rectilinear slide rule.

Like many of the other long-scale slide rules discussed in this paper, the ARC has provisions for making approximate calculations. A single short D-scale semi-circular scale runs near the outer edge of the disk for making calculations to about three significant digits before making more precise calculations on the longer expanded semi-circular scale. The ARC also has L, S, and T scales near the outside edge, and sets of S and T semi-circle rings nested amongst the long D-scale rings.

From the instructions [32], I know that the ARC was marketed by ADF Products of New York City, a company owned by Andrew D. Frankenfield. Frankenfield told me [33] that a commercial printer made his slide rules. He also patented a circular version of this slide rule. The patent model had a scale length of about 38m and two magnifiers on the cursors to help read the scales. It would have been the longest slide rule scale known if it had gotten into production. He also patented gridiron versions that spooled from one roller to another. The only known examples of the ARC *Extended Slide Rule* are in the possession of the designer and in my collection.

Discussion

Table I summarizes the VLS circular slide rules found. There are 14 in total. The Sutton is shortest and possibly the first of the VLS circular slide rules for which we have detailed data. The actual scale lengths range from about 2.8m to 14m, while the effective scale lengths range from about 4m to over 22m. VLS circular slide rules with scale lengths of 4m or more are capable of resolving at least four digits, while those with effective scale lengths greater than about 10m can resolve results to nearly five digits. The "Square" Atlas model has the longest effective scale length (22m) of all slide rules known, save for the 24m Loga cylindrical slide rule.

Spiral- and concentric-scale slide rules are the most size-efficient of all long-scale slide rules. One wonders why cylindrical slide rules were invented, given their cumbersome size and the very early introduction of the spiral and concentric scale forms in 1630. Perhaps for the same reason that most VLS circular slide rules were developed: the inventors did not know of any earlier VLS circular slide rule developments.

Acknowledgements

I should like to acknowledge the many contributors that responded to my requests for information on long-scale slide rules. Specifically for VLS slide rules, Bob Otnes sent me the Montaque Worman *C/10/LEA* circular slide rule to examine and figure out how it works; Charles Mollan sent details and images for the Lilly Extended slide rule; and Andrew Frankenfield sent an ARC Extended semi-circular slide rule for me to examine and keep for my collection. I should also like to thank the Science Museum in London for preserving the very early Sutton and Adams spiral slide rules, and providing pictures and details for these and the Dixon slide rules for publication in earlier editions of *JOS*. Readers are encouraged to visit the Science Museum when traveling to London to view these and many other interesting slide rules in their collection. Finally, I should like to express my appreciation to Peter Hopp for improving my knowledge of the Delamain and Fearnley slide rules, and for his kind review of this report.

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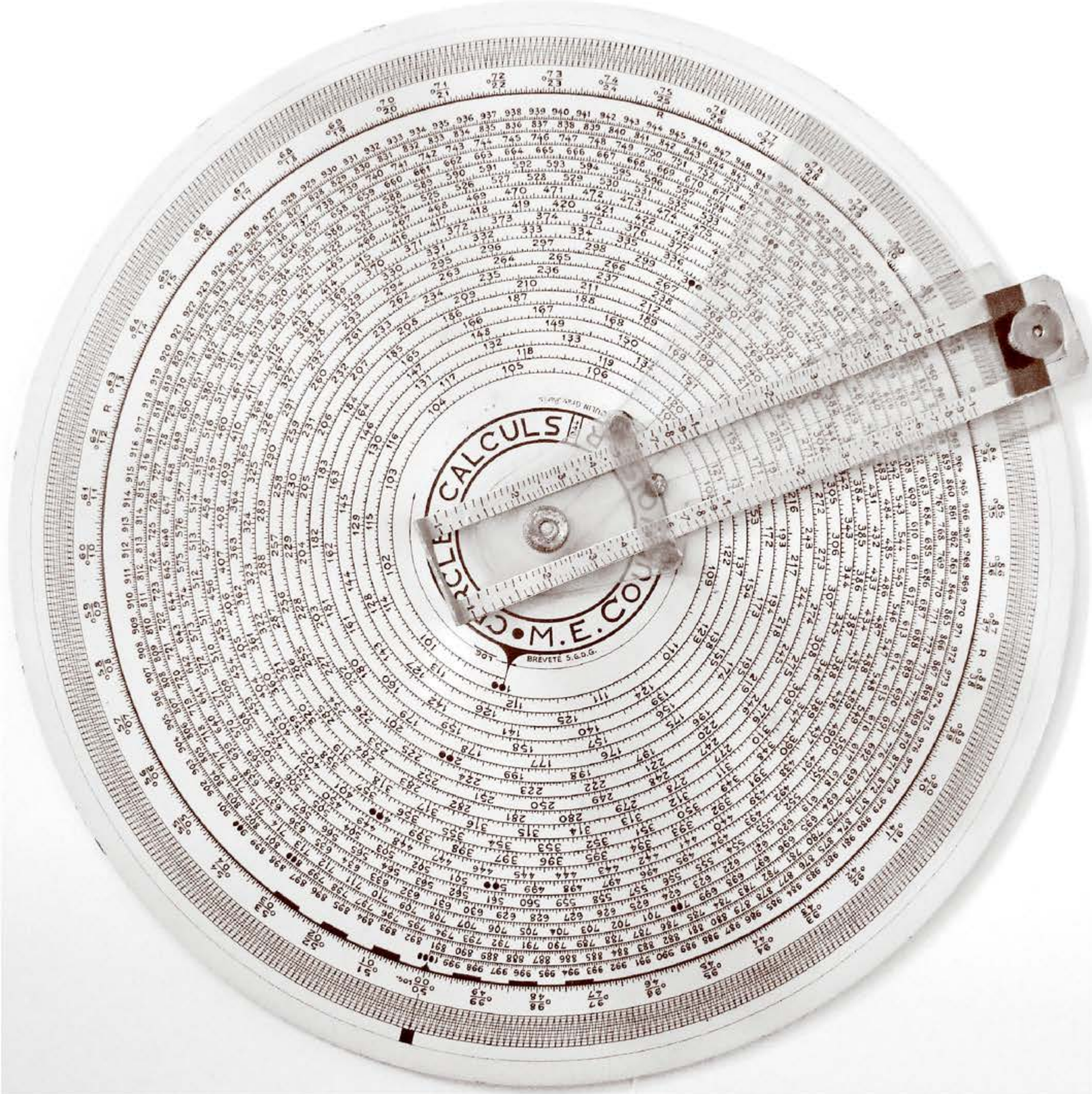

Figure 14. The Courvoiser Cercle à Calculs

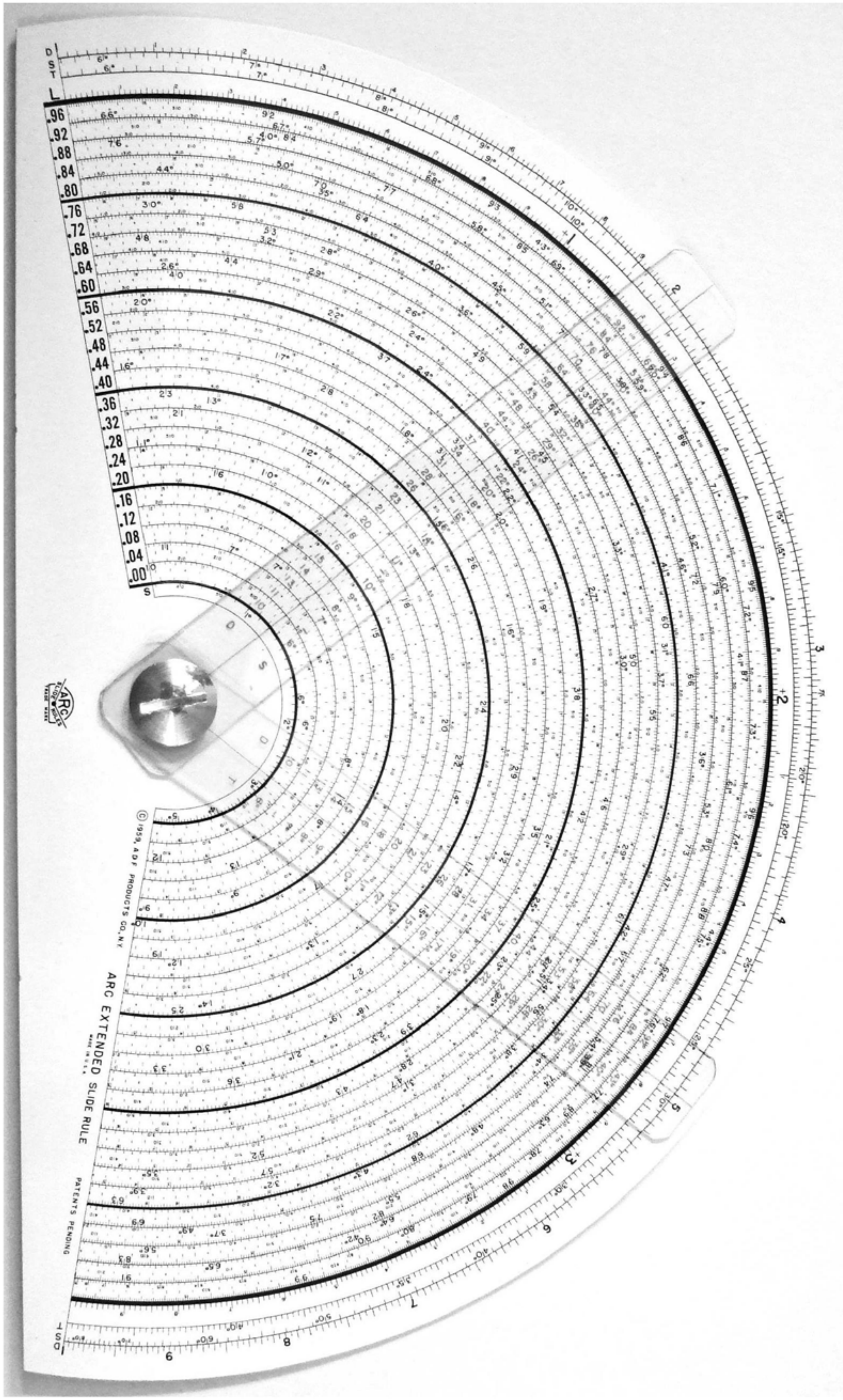

Figure 15. The ARC Extended Slide Rule