A Qualitative Analysis on a Quantitative Device

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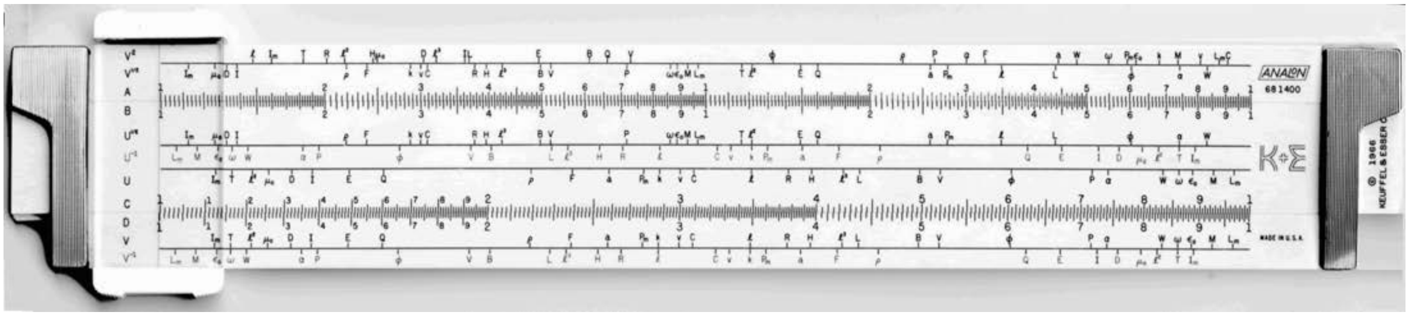

Figure 1. Front of Analon

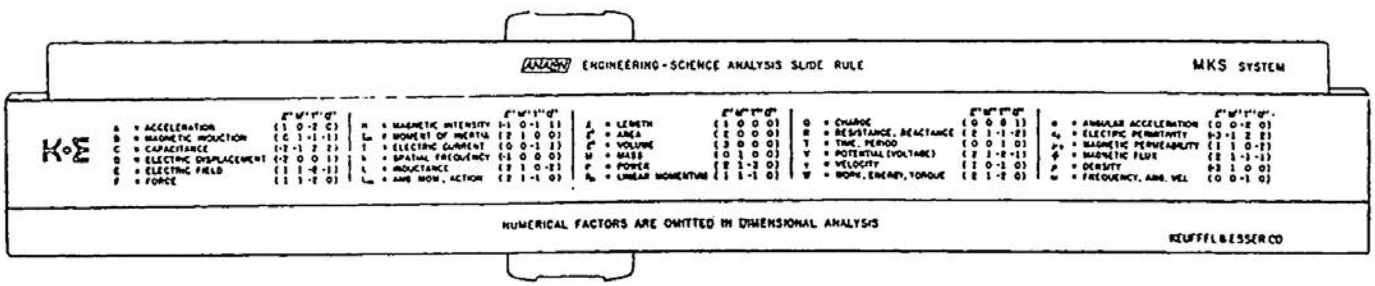

Figure 2. Sketch of back of Analon

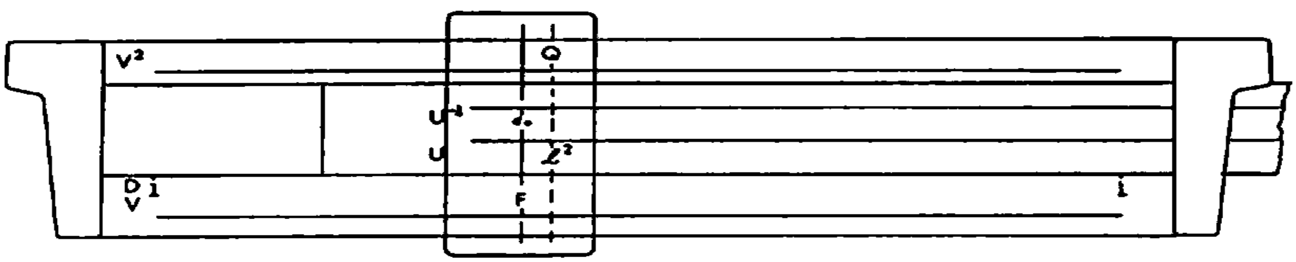

Figure 3. Using the Analon

A unique device, at the end of the slide rule epoch, was the Analon slide rule, slide rule Number 68 1400 by K&E, in 1966. This slide rule was different, especially when compared to the K&E Decilon and other manufacturers' slide rules of approximately the same time interval. It apparently did not sell as well as the Decilon. According to the K&E Analon manual, the purpose of the rule was to provide a tool "for dealing with physical qualities having both size and dimension".[1]

The Analon is one-sided, approximately 32cm long by 4.5cm wide by 5mm thick, with a legend on the reverse face, and weighing approximately 71 grams without its embossed leather case. See Figures 1 and 2. There were only eleven scales on the device, including the four scales found on most slide rules (the A/B and C/D scales). Of the eight remaining, four are related to the V scale and are located on the main body of the slide rule. They are V, the inverse of V (labeled V^{-1}), square root of V (labeled $V^{1/2}$), and the square of V (labeled V^2). The scales for U were located on the actual slide, as were the inverse of U (labeled U^{-1}) and the square root of U (labeled $U^{1/2}$). Both U and V inverse scales are labeled in red. On the Analon, physical dimensions such as length, time, charge, force, square root of force, and square of force were located on the U and V scales by thirty various symbols shown on the back of the rule.

There were four uses for the rule. The first was to check equations dimensionally. For example, verifying that resistance times charge divided by inductance equaled current. Second, the device was to be used as an

[1]Page one of *Analon Manual.*

aid in recalling formulas. For example, asking if capacitance equaled $\epsilon_0 r/4\pi$ or $r/4\pi\epsilon_0$. Third, the device could be used to derive formulas. For example, if a pendulum of length l accelerating had the form $T = Al^r g^s$, what were the values of r and s. Finally, the device could be used to perform numerical calculations, such as multiplication, division, squares, square roots, and other calculations using the A/B and C/D scales as on conventional slide rules.

Qualitative dimensions—such as length and time—without quantitative items could be applied. It should be noted that the V scale is typically in the numerator, and the U scale is in the denominator. For example, taking length on the V scale and lining it up with the T on the U scale gives velocity (v) on the V scale when we look at "1" on the C scale and move the hairline over to it. This internal consistency does help in recalling formulas. With dimensions, numbers are not included in the calculations. For example, a length that is doubled or tripled is still a length, and the square root of a mass is still a mass.

Let us approach a sample problem, such as determining the force of repulsion between two identical charges (Q) separated by a distance (l).

$$F = \frac{Q^2}{4\pi\epsilon_0 l^2}$$

This is how it is done. Push the hairline to Q on the V^2 scale. Move l^2 on the U scale under the hairline. Move the hairline to ϵ_0 on the U^{-1} scale. At the hairline read F on the V scale. See figure 3 below. As π is a number, it is left out of the calculation.

Let us approach another sample problem. To verify that resistance times charge divided by inductance equals current (as mentioned earlier), first move the hairline to R on the V scale. Move L on the U scale under the hairline. Move the hairline to Q on the U scale. On the V scale under the hairline we see the symbol I.

If a specific qualitative item was desired that is not among the thirty dimensions included on the back, the user was advised to scratch the mark with the point of a compass and fill in the indentation with ink[2]. For example, pressure is force (F on the V scale) divided by area (l^2 on the U scale) and gives the number '1967', which could be a scribed "Pr" mark on the V scale. Please note that space was provided in the manual at the bottom of Table 2 to write in the new symbol. See Table 2.

Table 1.

Symbol	Quantity	Basic Dimensions			
		Length	Mass	Time	Charge
a	Acceleration	ℓ		T^{-2}	
B	Magnetic Induction (Magnetic Flux Density)		M	T^{-1}	Q^{-1}
C	Capacitance	ℓ^{-2}	M^{-1}	T^2	Q^2
D	Electric Displacement (Electric Flux Density)	ℓ^{-2}			Q
E	Electric Field	ℓ	M	T^{-2}	Q^{-1}
F	Force	ℓ	M	T^{-2}	
H	Magnetic Intensity	ℓ^{-1}		T^{-1}	Q
I_m	Moment of Inertia	ℓ^2	M		
I	Electric Current			T^{-1}	Q
k	Spatial Frequency	ℓ^{-1}			
L	Inductance	ℓ^2	M		Q^{-2}
L_m	Angular Momentum, Action	ℓ^2	M	T^{-1}	
ℓ	Length	ℓ			
ℓ^2	Area	ℓ^2			
ℓ^3	Volume	ℓ^3			
M	Mass		M		
P	Power	ℓ^2	M	T^{-3}	
P_m	Linear Momentum	ℓ	M	T^{-1}	

Table 1.

Table 2.

Symbol	Quantity	Basic Dimensions			
		Length	Mass	Time	Charge
Q	Charge				Q
R	Resistance, Reactance	ℓ^2	M	T^{-1}	Q^{-2}
T	Time, Period			T	
V	Potential (Voltage)	ℓ^2	M	T^{-2}	Q^{-1}
v	Velocity	ℓ		T^{-1}	
W	Work, Energy, Torque	ℓ^2	M	T^{-2}	
α	Angular Acceleration			T^{-2}	
ϵ_0	Electric Permitivity	ℓ^{-3}	M^{-1}	T^2	Q^2
μ_0	Magnetic Permeability	ℓ	M		Q^{-2}
ϕ	Magnetic Flux	ℓ^2	M	T^{-1}	Q^{-1}
ρ	Density	ℓ^{-3}	M		
ω	Frequency, Angular Velocity			T^{-1}	

Table 2.

[2]Page ten of *Analon Manual*.

This can lead to potential problems. For example, if we take a wire of length, resistance, area, and conductivity, we can solve for conductivity, sigma (σ) with $\sigma = l/RA$. Taking l on the V scale, moving R on the U scale so that it is above it, and moving the hairline so that l^2 on the U^{-1} scale is listed gives a reading on the D scale of the number '762.' However, should the number '762' show up after a calculation, it does not always signify that conductivity is the final result.

In some ways, such a device can be useful. For example, the manual asks the user to perform a dimensional check on the integral forms of Maxwell's equation, Gauss's law (electric), Gauss's law (magnetic), Ampere's law, and Faraday's law of induction. However, they leave it up to the user to solve these[3].

Prior to 1960, systems were in the MKSC units (Meter, Kilogram, Second, Coulomb). After 1960, the MKSA system (or SI system for Systeme Internationale) was adopted (Meter, Kilogram, Second, Ampere). A conversion table was given for converting from the MKS system to the egs system, to the f lbm 5 system, and to the f lbf 5 system[4]. The Analon Rule is in the MKS system.

There are inherent problems with dimensionless analysis presented in the Analon. For example, what one symbol means to the creator of the slide rule may not be one we'd use today. In addition, we see in the book by Albert Shadowitz [2], that dimensionless analysis, using similar quantities, does not always come up with the same values (see Table 3, below, from Shadowitz [2], and compare it with Tables 1 and 2 to see the variants between the two systems). Discrepancies between the two tables include the values for capacitance, charge, inductance, magnetic flux, permittivity, resistance, and voltage. It can be seen that manipulation of some form is needed to reconcile the two tables.

The Analon does allow one to see quickly if the formula he is remembering is the true one, and to try to derive new formulas; however, the user should hang onto his other slide rules should he have any common or natural logarithmic functions and must produce a number for an answer.

Bibliography

1. Smyth, Michael. (1967) K&E Analon Engineering-Science Analysis Slide Rule Instruction Manual. New York, Keuffel & Esser Co.

2. Shadowitz, Albert. (1975) The Electromagnetic Field. New York, McGraw-Hill Co. (Reprinted 1988 by Dover Publications, Inc., Mineola, N.Y.).

Table 3.

This table lists the exponents $\alpha, \beta, \gamma, \delta$ to which the four basic dimensions—[m], [kg], [s], [A], respectively—must be raised in order to obtain the dimensions of various electromagnetic quantities. For example, the exponents for the dimensions of permittivity are listed as $-3, -1, 4, 2$; so $[\epsilon_0] = [m^{-3}\ kg^{-1}\ s^4\ A^2]$. Here, as elsewhere in the book, we distinguish between dimensions (e.g., length) and units (e.g., meter) by using a square bracket for dimensions. Thus, [m] means the dimension which has the meter for its unit. $[m^\alpha\ kg^\beta\ s^\gamma\ A^\delta]$

Capacitance	Symbol	Unit	α	β	γ	δ
Capacitance	C	F	-2	-1	4	2
Charge	q	C	0	0	1	1
Charge density	ρ	$C\ m^{-3}$	-3	0	1	1
Conductance	G	mho	-2	-1	3	2
Conductivity	g	$mho\text{-}m^{-1}$	-3	-1	3	2
Current	I	A	0	0	0	1
Current density	J	$A\ m^{-2}$	-2	0	0	1
D field	D	$C\ m^{-2}$	-2	0	1	1
Electric field	E	$V\text{-}m^{-1}$	1	1	-3	-1
Energy	U	J	2	1	-2	0
Energy density	u	$J\ m^{-3}$	-1	1	-2	0
Energy flux	S	$J\ m^{-2}\ s^{-1}$	0	1	-3	0
Force	F	N	1	1	-2	0
H field	H	$A\text{-turns}\ m^{-1}$	-1	0	0	1
Hertz vector	Π	V-m	3	1	-3	-1
Impedance	Z	Ω	2	1	-3	-2
Inductance	L	H	2	1	-2	-2
Length	ℓ	m	1	0	0	0
Linear charge density	λ	$C\ m^{-1}$	-1	0	1	1
Magnetic field	B	T	0	1	-2	-1
Magnetic flux	Φ	Wb	2	1	-2	-1
Magnetization	M	$A\ m^{-1}$	-1	0	0	1
Magnetomotance	$\mathfrak{M}$	A-turns	0	0	0	1
Mass	m	kg	1	0	0	0
Momentum	p	$m\ kg\ s^{-1}$	1	1	-1	0
Momentum density	g	$m^{-2}\ kg\ s^{-1}$	-2	1	-1	0
Momentum flux	f	$m^{-1}\ kg\ s^{-2}$	-1	1	-2	0
Permeability	μ	$H\ m^{-1}$	1	1	-2	-2
Permittivity	ϵ	$F\ m^{-1}$	-3	-1	4	2
Polarizability	α	$F\ m^{-2}$	0	-1	4	2
Polarization	P	$C\ m^{-2}$	-2	0	1	1
Potential	ϕ	V	2	1	-3	-1
Potential energy	U	J	2	1	-2	0
Power	P	W	2	1	-3	0
Poynting vector	S	$W\ m^{-2}$	0	1	-3	0
Reluctance	$\mathcal{R}$	$A\text{-turns}\ Wb^{-1}$	-2	-1	2	2
Resistance	R	Ω	2	1	-3	-2
Resistivity	ρ	Ω m	3	1	2	1
Surface charge density	σ	$C\ m^{-2}$	-2	0	1	1
Surface current density	j, J_ℓ	$A\ m^{-1}$	-1	0	0	1
Time	t	s	0	0	1	0
Vector potential	A	T	1	1	-2	-1
Voltage	V	V	2	1	-3	-1
Work	W	J	2	1	-2	0

Reference Table 3, reprinted from [2] with the permission of Dover Publications, Inc.

[3]Page 23 of the *Analon Manual.*

[4]Table IIA of the *Analon Manual,* Page 31.