Two Unusual Types of Trigonometric Scales

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Introduction

Standard trig scales are related directly to the main scales (C/D). Common usage is to set a chosen angle value on a trig scale and read the corresponding value on one of the main scales, or vice versa for the inverse values. Angle values can be in degrees/minutes/seconds (DMS) or in decimal values. Scales based on 400 degrees (grads), rather than the more common 360 degrees, are also found.

There are two systems that differ significantly from this "standard':

1. The application of "evenly spaced" trig scales, e.g., Australian W&G slide rules.

2. The application of "differential trigonometric scales", e.g., British Thornton slide rules.

Evenly Spaced Scales

The sine scale is "evenly spaced" with a range of 0° to 90°, while for the tangent scale, 0° to 45°, the corresponding values 0 to 1 are "evenly spaced".

Obviously, these scales do not relate to the C/D scales, but to their own matching scales.

The idea is, obviously, to avoid the awkward denseness of scales at one end, and the seemingly superfluous accuracy at the other end. In other words, to distribute the possible errors more evenly.

The disadvantage of this? The resulting data cannot be used for further calculations without transferring them to other scales.

Differential Scales

"Differential scales" work according to a more sophisticated principle:

$Sd(x)$ being "differential sinus": $Sd(x) = x/\sin(x)$.

And, $\sin(x) = x(\sin(x)/x)$.

Then, $\sin(x)$ is found by first setting the cursor to x on the D scale.
Bring x on the $Sd(x)$ scale under the cursor. Read $\sin(x)$ on D under the C index.

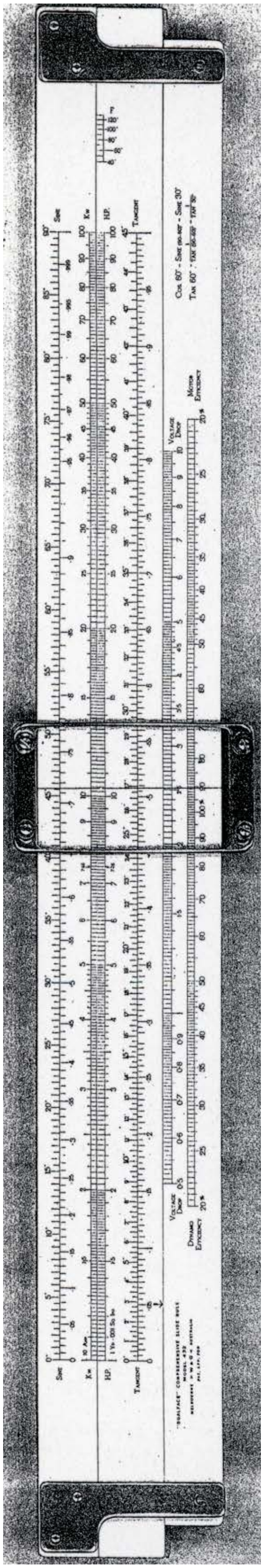
A similar procedure is applied for tangent or inverse values.

This system claims to have greater or equal accuracy over the entire range, compared with the "standard" layout.

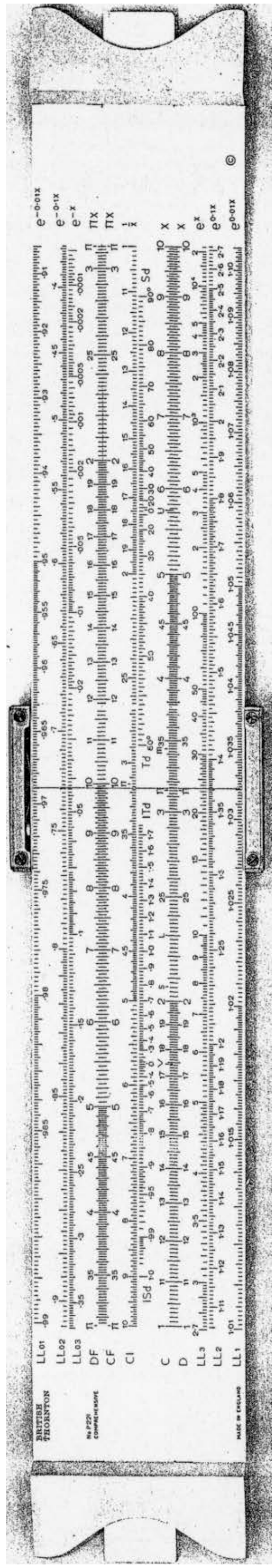
Why weren't these scales in general use? Three possible reasons are:

1. Patents.
2. The inconveniences in use were judged to outweigh possible advantages.
3. Possible lack of real advantages? (See test below.)

Australian W&G



British Thornton



The Test

Let us have a look at how these scales perform in practice, compared with the “standard” scales.

Taking departure in $\sin 85°$:

Here, the “even scale” shows that the result is pretty close to 0.996 (unnecessary lack of subdivisions).

The “difference scale” and the “standard scale” both show something in the neighborhood of 0.996.

The definite winner, however, is a rule with a P scale. Here we can safely say 0.9962.

Then we try for sin 5°: On the “even scale” we guess at 0.87 (again due to an unnecessary lack of subdivisions).

On the “difference scale’, 0.87.

On the “standard scale” (extended ST), 0.872.

In the middle of the sine scale, the different systems seem to give more or less equal results.

Similar testing for tangent shows the same tendency.

The “even scale” layout apparently has one small advantage: the scales extend down to 0°, rather than to c. 5.7°.

Sine scales related to A/B scales (e.g., K&E) extending the range down to c. 35′ have not been considered here, although they are excellent for small angles, because they work to their best advantage only on a 20-inch rule, where the right half would be of normal 10-inch length.

Neither has the use of gauge marks / and // been considered. They do not seem to have any significant effect on our comparisons, and are not found on all rules.

Conclusion

The standard trig layout, especially if it has a P scale, two T scales plus extended scales, offers the overall best results, and clearly the simplest procedures, compared with these “odd” scales.

PS. The Faber-Castell 2/84(N) Mathema, with its extreme scale extensions, reads down to $\sin / \tan 3.8°$ and up to $\tan 55.8°$ (degrees converted from grads).