Slide Rule Decimal Point Location Methods

Edwin Chamberlain

Background

We use the decimal point as the separator between the whole and the fractional parts in numbers. However, it has not always been that way. The decimal point first came into use as a separator in the early 1600s. Napier, the inventor of logarithms, began using the decimal point about 1616 [1]. Prior to that time, a number like 6.76 might have had the whole and fractional parts separated as such: 6|76. According to Cajori [1], that is how William Oughtred, the inventor of the slide rule, would have done it. Since the time of Napier and Oughtred, the decimal point has become the standard symbol for marking the place where the fractional part of a number begins.

Napier's and Oughtred's works left us the great legacy of the slide rule. The problem with the slide rule, however, is that it does not give the position of the decimal point, except for operations involving certain scales such as the log log scales. The slide rule only provides the sequence of digits in the answer to a calculation, with no information about the position of the decimal point. This was a vexing problem for some slide rule users in the past, especially for casual users and for students first learning how to use a slide rule. In 1910, Pickworth [2] wrote in the preface to his book, *The Slide Rule,* that "...while most practiced users experience no difficulty in estimating the magnitude of the result by inspection or rough calculation (which gives the decimal point location), others find rules (for locating the decimal point) of considerable assistance."

Because many users of slide rules have found it difficult to fix the location of the decimal point, numerous methods and devices have been developed to help do this. This paper surveys various methods proposed and used. Two categories of methods were found, one based on mental operations, and the other based on the manipulation of special scales or other devices on a slide rule. These two different categories of methods for determining the position of the decimal point can be thought of as one, "mental" and two, "assisted" methods: the mental procedures being the methods requiring some ability to visualize the problem, and the assisted methods having some sort of physical crutch. This report concentrates on the problem of determining the location of the decimal point when using the primary C and D scales for multiplication and division. Much of the information on placing the decimal point was obtained from slide rule collectors, journal articles, conference proceedings, instruction manuals, catalogs and books, some directly from markings on slide rules, and more from researching patent documents.

Mental Methods for Locating the Decimal Point

There are numerous "mental" methods for determining the position of the decimal point in the result of a slide rule calculation. I have separated these into three types: a) inspection; b) approximation; and c) digit count. The first type - inspection - requires only a quick glance at the problem to know the order of magnitude of the answer (and, thus, the location of the decimal point). The second type - approximation - needs a little more mental exercise, and the third type – digit count – requires a more rigorous analysis. The various methods are discussed in detail below.

Inspection

The decimal locating methods in this category only require a brief look at the problem. Often the problems are simple, for example, a problem like 3.5×7.6. From the slide rule we find that the product of these two numbers contains the digits 266. One easily concludes that the answer is less than 100 and more than 10 just with a quick look. The product, thus, is 26.6, not 2.66 or 266. Breckenridge [3] described this as a "mental survey" method, requiring only a look at the factors involved in the problem to determine the order of magnitude of the result. This method works well for problems with factors having 1 or 2 digits to the left of the decimal point. Using a little different argument, Hartung [4] suggested that "common sense" be used in determining the correct answer. One knows that if the calculated speed of an airplane comes out to be 488 on the slide rule, that the answer is 488 miles per hour, not 48.8 mph or 4880 mph. Most airplanes do not stay in the air at 48.8 mph, nor do they fly at supersonic speeds of 4880 mph. The Post Versatrig Slide Rule manual [5] suggests the same approach. Thacher [6] made a similar suggestion, writing that the answer to many problems is based on "prior knowledge" of the expected range of values.

It is important to note that inspection methods require the same mental facility that engineers, scientists, accountants, etc. need to be successful in their work. Thus, for most practiced professionals, the inspection method comes naturally. However, for some problems there can be too many factors, and the numbers can be too large or too small. These problems can tax one's mental facilities, and thus more rigorous approaches such as one of those employing "approximation" or "digit counting" may be needed.

Approximation

Approximating the answer in a slide rule calculation is a step up in complexity over just inspecting the problem. It usually entails mentally rounding off the factors, and making a rough mental calculation to determine the appropriate order of magnitude for the result. For instance, the problem 32.6 × 58.9 becomes 30 × 60 in the mind, and the approximate answer is 1800. From the slide rule one finds that the result contains the successive digits 192. The approximate calculation tells us that the decimal point is located to the right of the 4th digit from the left. We, thus, find that the answer to the problem is 1920. Most of the instruction manuals and books reviewed suggest this "approximation" method for locating the decimal point. Numerous different phrases are used to describe this method. For instance, Breckenridge called it "rough calculation", Cullimore [7] "rough check", Hartung [4] "estimate", Hills [8] "approximate problem", Johnson [9] "round off", Kells, *et al* [10] "rough calculation", Stanley [11] called it "mental multiplication", and Asimov [12] considered it a "substitution of similar numbers".

I have found that most engineers who used the slide rule regularly employed the inspection and approximation methods, as these methods fit neatly with the engineer's training to visualize problems. However, many problems are complex enough that even the best of us are challenged mentally to determine the order of magnitude of the result. For these problems, one either reverts to pencil and paper approximations, or relies on a more rigorous method of counting and keeping track of digits.

Digit Counting

Digit counting methods require some systematic method of keeping track of the number of digits to the left or right of the decimal point. This is the most exacting of the mental methods. There have been many different approaches to counting the number of digits. All rely on some orderly method of counting and adding digits, and some other basic observations. Later I will discuss slide rule accessories for keeping track of the digit count, but here I will present methods that ordinarily can be done mentally. These digit-counting rules apply to the use of the single-cycle C and D scales.

Cox Characteristic or Digit Count Method. The earliest decimal-locating method found is based on the characteristic of a number. In his early 1900s instruction manual for K&E slide rules, Cox [13] defined the characteristic as the number of digits in the integral part of a number (that part of the number having integers). Fractional numbers have negative characteristics equal to the number of places to the right of the decimal point to the first non-zero integer. The characteristic of 462.0 is 3 and the characteristic of 0.00623 is -2. To use this method, one must remember that for multiplication problems the characteristic of the product is equal to the sum of the characteristic values of the two factors, if the calculation is made with the slide projecting to the left. If the slide projects to the right, then adjustments must be made to the characteristic of the result by subtracting 1.

Example No. 1

$$462.0 \times 0.00623 = [288]$$

(slide rule reading with slide projecting to the left)

The characteristic of 462 is 3, and the characteristic of 0.00623 is -2; the sum of the characteristics being +1. No adjustment in characteristic of the product is needed, as the slide projects left.

The result, thus, is 2.88, the characteristic of the product being +1.

Example No. 2

$$462.0 \times 0.0021 = [97]$$

(slide rule reading with the slide projecting right)

The characteristic of 462 is 3, and the characteristic of 0.0021 is -2; the sum of the characteristics being +1. The slide projects right, therefore the characteristic is adjusted by subtracting 1 from the sum of the characteristic values, the resulting characteristic being +1 -1 = 0. The product, thus, is 0.97, the characteristic of the product being 0.

For division problems, the rule is to subtract the characteristic of the divisor from that of the dividend, and to add one if the slide extends to the right. Poland [14] also described this method for the Engineering Instruments slide rules, and Bishop [15], in his definitive slide rule instruction book, called the characteristic the "span".

Pickworth Digit Count Method. Also in the early 1900s, Pickworth [16] gave a similar set of rules for Faber slide rules, but defined the number of digits or zeros in a number as the basis for determining the location of the decimal point. The digit and zero count method is equivalent to the characteristic count method described by Cox [13]. However, Pickworth used a different approach to account for the adjustment factor. He used the position of the result in relation to the first factor in a calculation. For multiplication problems, if the result is obtained to the left of the first factor (the slide extends to the left), no adjustment in the digit count of the product needs to be made. If the result falls to the right of the first factor (the slide extends to the right), then the digit count of the product needs to be reduced by 1. Pickworth's rule for multiplication follows the same logic as Cox's rule. However, we will see that there are serious differences in the rules for division that could lead to confusion. Pickworth's rule for determining the adjustment factor for division problems is that: if the quotient is obtained with the result appearing to the right of the first factor (slide extending to the left), no adjustment needs to be made. If the result appears to the left of the first factor (slide extending to the right), then one is to be added to the total digit count. In the rules for adjusting the digit count of the result for multiplication problems, the direction of the result and the slide extension are the same, but for division problems they are in opposite directions. One, thus, needs to be careful in knowing what to watch, the position of the result or the direction of the slide extension. It is interesting to note that Edwin Thacher [6] described essentially this method for use on his cylindrical slide rule.

Modified Pickworth Digit Count Method. Pickworth [2] also presented another variation of keeping track of the decimal point, one not dependent on the direction of the result on the slide rule scale, or the direction of slide extension. Pickworth's rule is: "When the first significant figure in the product is smaller than either of the factors, the number of digits in the product is equal to the SUM of the digits in the two factors. When the contrary is the case, the number of digits in the product is 1 less than the sum of the digits in the two factors. When the first figures are the same, those (figures) following must be compared." Thus for our example No. 1 problem:

$$462.0 \times 0.00623 = [288]$$

(decimal point not located)

the first digit of the product is 2, which is smaller than the first digits (4 and 6) in either of the factors; therefore the digit count for the product is equal to the number of digits minus the number of zeros, which is: 3-2 = 1. The result, thus, is: 2.88.

For our example No. 2 problem:

$$462.0 \times 0.0021 = [97]$$

(decimal point not located)

the first digit of the product is 9, which is greater than the first digits (4 and 2) of both of the factors; therefore the digit count for the product is equal to the number of digits minus the number of zeros minus 1, which is: 3-2-1 = 0. The result thus is: 0.97.

For division problems, Pickworth's rule is "When the first significant figure in the DIVISOR is greater than that in the DIVIDEND, the number of digits in the quotient is found by subtracting the (number of) digits in the divisor from those in the dividend. When the contrary is the case, 1 is to be added to this difference. When the first figures are the same, those (figures) following (it) must be compared."

Richardson Places Method. In his 1918 instruction manual for Richardson slide rules, Richardson [17] added a little more clarity to the understanding of the placement of the decimal point. He wrote: "In multiplication it is not possible to get more places in the product than the sum of figures (places to the right of the decimal point) in both numbers, and it is not possible to get less places than ONE LESS than (the number of places in) this product." That explains why we sometimes need to subtract 1 from the sum of the place (digit) counts for the two factors.

We can think of the problem as if we are working with a 2-cycle set of calculating scales, like the A & B scales normally found at the upper margin of the slide. If we set the left index of the slide under the first factor (on the A scale), and then set the cursor hairline over the second factor (on the B scale), we will read the result under the hairline on the A scale. If the result falls within the limits of the first cycle on the B scale, we subtract 1 from the total place count. If the result falls within the second cycle, we do not make a correction. The slide is extending to the right for both cases. Now, if we move to the C and D pair of single-cycle scales to make the same calculation, we note that if the result falls to the right of the first factor on the D scale, we need to subtract 1 from the total place count. The slide is extending, in this case, to the right. Here we note that we do not have a second cycle to calculate on. Thus, if the second factor causes the hairline on the cursor to fall beyond the end of the D scale, we need to reverse the direction of the slide and place the right index of the C scale over the first factor on the D scale. We, then, find the result on the D scale to the left of the first factor. What we have done is to cause the result to fall on a second cycle, but in this case it is a "virtual" scale. For this case, the slide is extending to the left. Thus, we do not need to make a correction to the total digit count.

In describing his method of locating the decimal point, Richardson used the word "figures" to describe what Cox called the "characteristic" of a number, and what Pickworth called "digits". Richardson followed the same approach as Cox in determining the adjustment factor. However, he described a little different approach for numbers containing decimal points. He stated that: "If there is a decimal point in either (the) multiplier or multiplicand, or in both, first treat them both as if they were whole numbers . . . and then move the decimal point of the product to the left a number of places equal to the sum of the decimal places in both the multiplier and multiplicand." Richardson's solution to our example No. 1 multiplication problem is as follows:

$$462.0 \times 0.00623 = [288]$$

(slide rule reading with slide projecting to the left)

$$462 \times 623 = [288]$$

(factors treated as whole numbers)

The number of places in 462 is 3, and the number of places in 623 is 3, the number of decimal places moved is 5 (all in the second factor). There is no adjustment in number of places in the product, as the slide projects left. Thus the number of places in the product is 3+3-5 = +1. The answer, thus, is 2.88, the number of places in the product being +1.

Pickworth and Richardson also give rules for placing the decimal point when using the A and B pair of scales. The details of these methods will be left to the reader, but one should note Richardson's warning that: "the placing of the decimal point (when using the A and B scales) . . . is rendered somewhat more difficult by the fact that the scales are half length and it is consequently more difficult to lay down any rule for counting the slide projections."

Slater Decimal Point Shift Method. Slater [18] described a method that he calls "shifting the decimal point". The purpose of this method is to make it easier to do mental multiplication and division by making one of the factors to have only one place. For a product of two factors, the decimal point of one of the factors is moved a certain number of places in one direction to make the value of that factor greater than or equal to 1, but less than 10; i.e., make it have only one place. It follows that the decimal point of the second factor must be moved the same number of places, but in the opposite direction. For our example No. 1 problem, the decimal point is located as follows:

$$462.0 \times 0.00623 = [288]$$

(slide rule reading)

$$462.0 \times 0.00623 = 4.62 \times 0.623$$

Note that the decimal point was shifted two places to the left in the first factor, and two places to the right in the second factor. For division problems, the decimal point is shifted in the same direction in both the numerator and denominator.

Stanley Characteristic Method. W.F. Stanley [19] gave one more version of the characteristic method for the Fuller cylindrical calculator. This time the characteristic of a number is defined differently. It is defined as if one were determining the characteristic of the logarithm of a number. He wrote that the characteristic is "...the number of figures before (to the left of) the decimal point, minus 1". Thus the characteristic of 462.0 is 3 - 1 = 2 and the characteristic of 0.00623 is -2 -1 = -3. Stanley also gives rules for adjusting the characteristic of the product, but these rules apply to the special pointer indicators on the Fuller spiral slide rule. They are, however, equivalent to stating that if the slide extends to the left, 1 is added to the sum of characteristic values, and if the slide extends to the right, there is no adjustment to the characteristic of the product. Again, for our example No. 1 problem, the decimal point location is found as follows:

$$462.0 \times 0.00623 = [288]$$

(reading with slide projecting to the left)

The characteristic of 462 is 2, and the characteristic of 0.00623 is -3. The answer, thus, is 2.88, the characteristic of the sum being 2 - 3 +1 = 0.

For our example No. 2 division problem, the decimal point location is found as follows:

$$462.0 \times 0.0021 = [97]$$

(slide projecting right)

The characteristic of 462 is 2, and the characteristic of 0.0021 is -3; the sum of the characteristics being -1. The slide projects right, therefore the characteristic is not adjusted. The product, thus, is 0.97, the characteristic of the product being 0.

For division, Stanley's rule is to subtract the characteristic of the divisor from that of the dividend, and to subtract one from the characteristic if the slide extends left.

Scientific Notation or Standard Number Method. A commonly used digit-counting method is based on the scientific notation or standard number method of writing numbers. We remember that the scientific notation method writes numbers as a one-place number times some power of ten, e.g., 342.8 becomes 3.428×10^2, or 0.03428 becomes 3.428×10^{-2}. The decimal point is always placed just to the right of the first non-zero digit of a number, and then it is multiplied by the appropriate power of ten to give the true number value. Zanotti [20] and Slater [18] gave very good explanations for using the scientific notation, as did Hartung [21], Ellis [22] and Hemmi [23]. The key to this method is that when two numbers expressed in scientific notation are multiplied, the exponents of the powers of ten are added, and when one number is divided by another, the exponent of the power of ten of the divisor is subtracted from the exponent of the dividend. The powers of ten are added or subtracted, as appropriate, and multiplied by the slide rule reading. For instance, for the problem:

$$462 \times 0.00623 = [288]$$

(slide rule reading)

Using the scientific notation method in concert with the approximation method, we rewrite the problem as:

$$4.62 \times 10^2 \times 6.23 \times 10^{-3}$$
$$\approx 5 \times 10^2 \times 6 \times 10^{-3}$$
$$\approx 30 \times 10^{-1}$$
$$\approx 3$$

The result, then, must be 2.88, which is approximately equal to 3.

For division, the power of 10 exponent of the divisor is subtracted from that of the dividend to obtain the power of 10 for the quotient. The scientific notation method may appear to some to be not easily facilitated without writing down the approximate results and the exponents. However, many experienced slide rule users can readily keep track (in the mind) of both the approximate value and the exponent parts for most multiplication and division problems. One can, of course, always resort to pencil and paper to keep track of these values, if necessary.

Clason 10s Count and Place Count Method. Clason [24] was a slide rule collector and enthusiast who set out to improve the understanding of using the slide rule. His law for locating the decimal point in multiplication is based on "10's counts" and "place counts". The "10's count" is equal to the number of digits minus 1 for numbers greater or equal to 1. The place count is equal to the number of digits, or the 10's count plus 1. For numbers less than 1, the 10's count is equal to the place count, which is equal to the number of places from the decimal point to the right of the first non-zero digit. To determine the 10's count of the product in multiplication, the 10's counts of the two factors are added. If the right index of the slide is set on the first factor, then 1 is added to the 10's count to get the place count. If the left index is set on the first factor, then 2 is added to get the place count of the product. Clason's rule becomes a little more complicated if the sum of the 10's count is -1 or less. For that case, the place count of the product is equal to the 10's count if the right index is used, and it is equal to the 10's count plus 1 if the left index is used. It is clear to me that one could easily be confused in trying to follow Clason's method.

Assisted Methods for Locating the Decimal Point

Many different physical aids have been developed to assist the slide rule user in determining the position of the decimal point. The simplest just provides instructions on the slide rule. Some of these aids involve special scales or digit-registering cursors. Others employ accessory devices. For the purposes of this discussion, we will break down this category into several sub-categories. They are: a) instructional aids; b) special scales; and c) special devices.

Instructional Aids

Some slide rules have digit count instructions placed directly on one side. These instructions often are abbreviations of the rules that have been discussed. Some take the form of cryptic markings, others more complete instructions or tables.

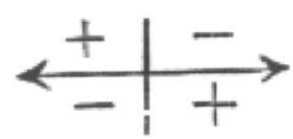
Slide Rules With Cryptic Scale Markings

Maker	Model No.	Slide Shift Mnemonics	Digit Count Mnemonic
Faber	367, 368	QUOT +1 & PROD -1	No
Faber-Castell	364, 367, 368	Quot. +1 & Prod -1	Yes
Hemmi	10" & 20" Mannheims	quot +1 & prod -1	Yes
Nestler	23	Q +1 & P -1	No
Post	1440, 1441	Quot. +1 & Prod. -1	Yes
Simplon	SR-2	Q+1 & P-1	No
Webber	4281S	Q +1 & P -1	No

Cryptic Markings. Some slide rules are marked on both ends of the face with cryptic markings. The marking on the right end of the lower stator is usually something like: Prod.-1 and the lower left stator: Quot.+1. The Prod.-1 marking is a reminder that if during a multiplication operation the slide extends to the right, that the decimal point in the product (after summing the number of digits in the multiplicand and multiplication) must be shifted 1 decimal point to the left (the digit count is decreased by 1). This is just like we learned earlier in the discussion of the digit counting method. Similarly, the Quot.+1 marking means that if the slide extends to the right in a division operation, the decimal point is shifted one digit to the right (the digit count is increased by 1).

These markings take different forms on different slide rules. For instance, on some Faber slide rules they appear as Prod.-1 and Quot.+1 or as PRODUCT -1 and QUOTIENT +1; whereas on some Nestler slide rules they appear as Q+1 and P-1. I have identified several slide rule makers and models with these markings, and have listed them in the table above. Note that I found no American made slide rules with these markings. Some makers also included a special mnemonic sign on the ends of the upper stator. Figures 1c shows this marking in the right corner of the Hemmi slide rule. The purpose of the mnemonic sign is to remind the user of the rules for the digit counting. The two arrows indicate the direction of decimal shifting in setting up the problem. The signs in the upper two quadrants indicate the sign of the digit (or zero) count for numbers in the denominator of an operation, while the signs in the lower two quadrants indicate the signs of the digit (or zero) counts for the numbers in the denominator. For instance, if in counting the digits, the decimal point is shifted to the right in a factor in the numerator, then the digit count is added; if it is shifted to the left (as it would be for a number less than 1), then the digit count is subtracted. The reverse holds for numbers in the denominator. Slide rules that I found

Special Mnemonic Sign:



with the digit count mnemonic are indicated in the table on the previous page. It appears that Faber first introduced these special scale markings, as they first appear in the early Faber instruction manuals by Pickworth [16]. Nestler, Hemmi and Post, and Simplon may have copied Faber's initiative.

Direct Instructions. Other slide rules have decimal locating instructions printed on one face. They are usually printed on the back face of the slide rule. For instance, K&E made a 10-inch Mannheim style slide rule with detailed instructions on the reverse side (Fig. 1) in the place where the table of constants and conversion factors usually go. This is an unusual K&E slide rule not included in any of K&E's catalogs. These instructions direct the user to find the decimal point by the Characteristic method. There is a table showing examples of determining characteristics from -2 to +3, and instructions for determining the decimal point shift based on the projection of the initial index as each new term is set on the slide. Directions are provided for both multiplication and division, and for using folded scales and half-length scales.

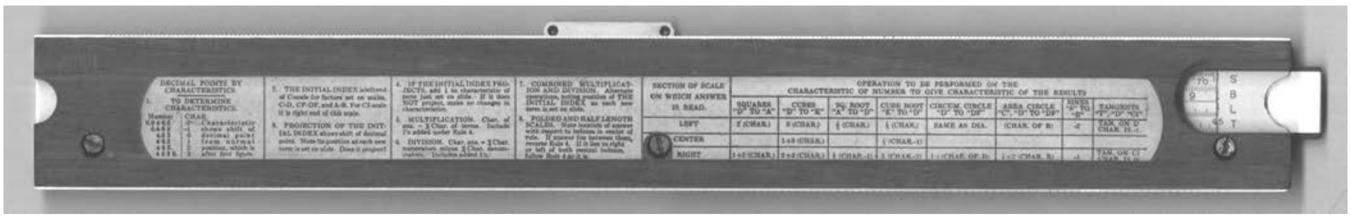


Fig. 1. Instructions on the back
of the special K&E slide rule.

A table summarizes the "Operation to be Performed on the Characteristic of a Number to Give the Characteristic of the Results" for various functions, including: squares, cubes, square root, cube root, circumference of a circle, area of a circle, sines, and tangents. It does not appear that K&E's attempt at providing help with finding the position of the decimal point was very commercially successful, as few examples of this slide rule are known, and K&E never gave it a model number, nor listed it in their catalogs.

Pickett & Eckel also included instructions for determining the decimal point location on their early slide rules. For instance, the early Deci-Log Log models (No. 2 & No. 3) had instructions (Fig. 2) for determining the position of the decimal point by the "digits and zeros" method. Rules are given for the sine, tan, and exponential functions. However, there is no guidance for ordinary multiplication and division. Later Pickett slide rules, including Models N901 and N902, also have instructions based on the "digits and zeros" method, but these instructions are also limited to finding the decimal point for certain functions, and not for simple multiplication and division operations.

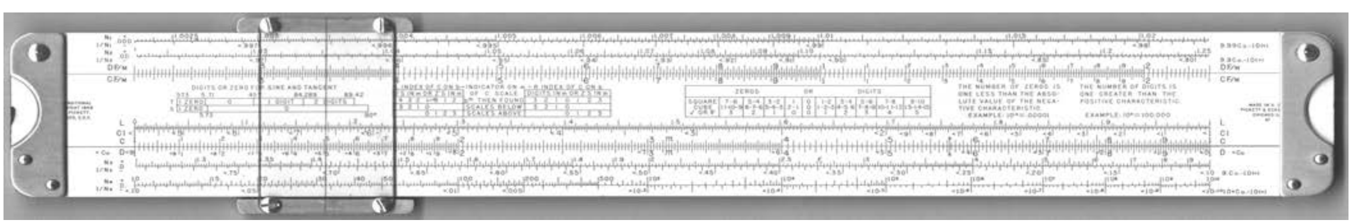


Fig. 2. Instructions on the back of the
Pickett & Eckel Deci-LogLog #2 rule.

Bishop [15] recommended that the formulas for determining the digit count for multiplication and division be marked in the upper left and right hand corners of the slide rule. I have not seen these markings scratched into slide rules, but have seen similar abbreviated rules penned on slide rule cases.

Special Scales

I found four different scale systems designed to provide the location of the decimal point for multiplication and division operations: 1) the Charles Hoare scales; 2) the System Wern scales; 3) the Eckel Deci-Point scales; and 4) the Kamm Decimal-Keeping scales. The Deci-Point and Decimal-Keeping scales are accessory scales whose only purpose is to locate the decimal point. Two different calculations must be made on the slide rules that have these scales; one calculation to determine the significant digits, and the other to determine the decimal point location. In contrast, the Charles Hoare and System Wern scales are direct reading with only one calculation required to obtain both the significant digits and the decimal point location.

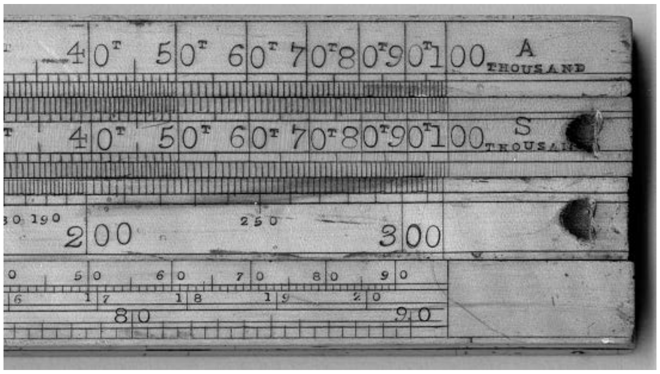


Figure 3. This rule by Hoare is about 32.3 inches long. There are six full cycles on the A and S scales. At the left end, they start at 0.1, while at the right end (shown) they end at 100,000.

The Charles Hoare Scales. Charles Hoare developed the earliest decimal point-reading scales that were found in this study. Hoare [25] included a cardboard slide rule (with two slides) inside the back cover of his book. Hoare's slide rule has a pair of 5-cycle scales at the top margin of the upper slide. The range of these scales is from 0.1 to 100,000. As long as the factors and the result fall within this range, this slide rule is direct reading. However, the scales are only 5" long, with each cycle being one inch long. The number of significant digits is limited to only one or two places. Bob Otnes has another version of the Hoare slide rule (Fig. 3) that has 30" long scales, each cycle having a length of 6". This slide rule, thus, has a direct-reading scale with a range of 0.1 to 100,000, and a resolution of two to three significant digits over most of its scale length.

The System Wern Direct Reading Scales. Carl, George, and Lars Wern [26] patented a two-disk circular slide rule in 1968. The scales are divided such that the range extends over many log cycles, much like the scales on the Hoare slide rule. Separate scales are provided for multiplication, division, roots, powers, etc. For multiplication the decimal point is direct reading for the range of 0.001 to 1,000,000, or eight cycles on a scale with a length of about 15". Each cycle has a length of about 1.8" and a resolution of two to three digits. The "System Wern" patent was employed by the IWA Company in Germany. Two IWA models are known to have these scales. Schure [27] described these scales on IWA model No. 1633 (Fig. 4) and Riehle [28] reported that No. 1638 has the Wern direct reading scales. Riehle also reported that the System Wern scales were used on a LOGOMAT circular slide rule.

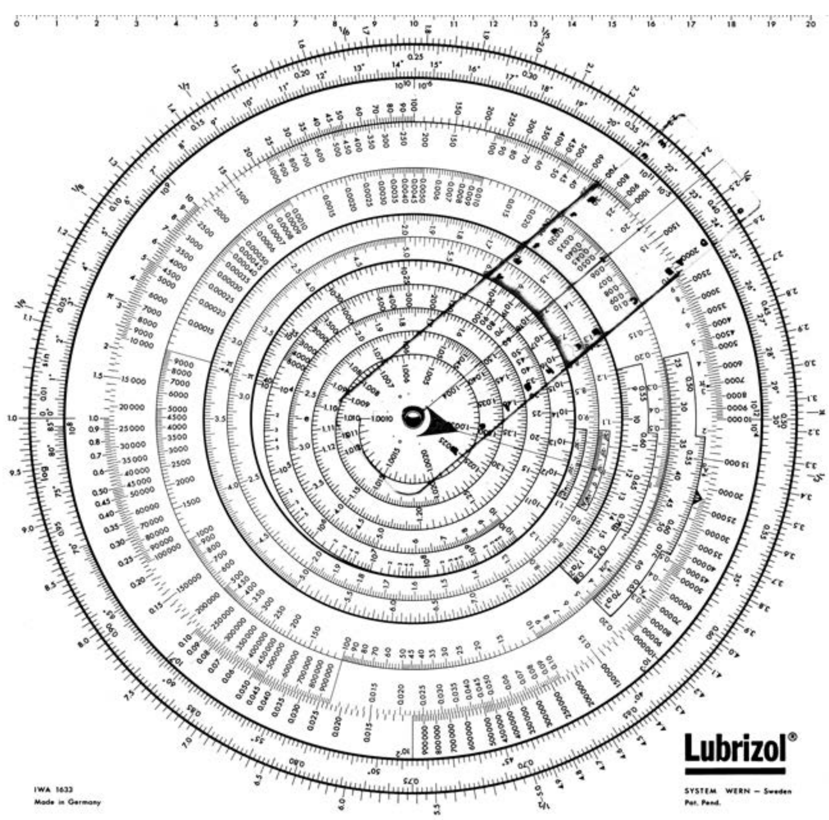


Fig. 4. Direct-reading scales on
IWA System Wern #1633 circular rule.

There seems to be no awareness by the Werns of the Hoare direct-reading scales. The Wern patent does not mention Hoare, nor his slide rule. Perhaps this is because the Hoare slide rule is quite rare, and not well known. I only found out about the Hoare slide rule when the editor of this journal brought it to my attention after I had submitted my copy for this article.

The Eckel Deci-Point Scales. Arthur F. Eckel developed the earliest decimal point-reading scales. These scales were copyrighted [29] in 1945, and a patent [30] was awarded in 1949. These scales appear only on the first Pickett & Eckel slide rule (Fig. 5), the one that is commonly referred to as Model No. 1, the Deci-Point slide rule.

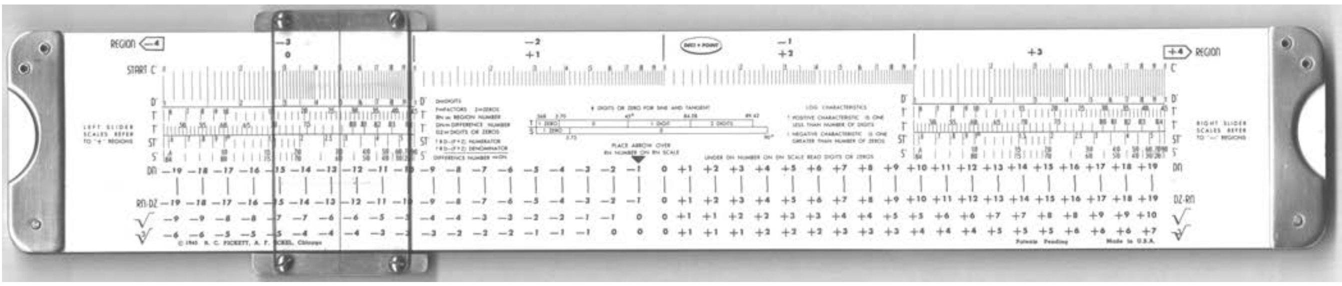


Fig. 5. Pickett & Eckel Deci-Point slide rule.

The special scales on this slide rule follow an algorithm based on the "digit and zeros" count method and an equation developed by Eckel:

$$DN = Dn - (Fn + Zn) - [Dd - (Fd + Zd)] - 1$$

Where:

DN is the difference number, or number of digits (or zeros) in the result,

Dn is the total number of digits (to the left of the decimal) in the numerator,

Dd is the number of digits (to the left of the decimal) in the denominator,

Fn is the number of factors in the numerator,

Fd is the number of factors in the denominator,

Zn is the number of zeros (to the right of the decimal) in the numerator,

Zd is the number of zeros (to the right of the decimal) in the denominator.

In addition to accounting for the "digits and zeros", the Eckel system keeps track of the contribution of each additional factor in the problem, and the direction that the slide extends. Instructions for using this slide rule were found in a Pickett & Eckel manual by Hartung [31]. The front of the slide rule has many of the common slide rule calculating scales, including: the C and D scales for multiplication and division; tan, sin, and log scales; and square and cube root scales. The reverse side (Fig. 5) has the special decimal point-locating scales. There is a four-cycle calculating scale at the upper slide margin on the stator, the cycles marked as regions from 0 to +4, from left to right, for multiplication operations, and from 0 to -4 from right to left for division problems. Directly below on either end of the slide, there are one-cycle scales that match the lengths of each of the four cycles on the stator. These scales are used to keep track of the digit count increase (or decrease) during multiplication (or division) calculations. If the result falls in the next cycle to the right, then the order of magnitude has been increased by 1, and thus the digit count is increased by one. The reverse is true if the result falls in the previous cycle. Multiplication problems are started at the left end of the slide rule, and division at the right end. At the lower margin of the slide, there are two scales (one on the slide and the other on the stator) marked in even intervals from -19 to 0 to +19. To complete the decimal point determination, a pointer at -1 on the slide DN scale is set opposite the region count (determined from the upper set of scales) on the stator RN-DZ scale. The number of factors and total number of zeros (to the right of the decimal) in the problem are then mentally subtracted from the total number of digits (to the right of the decimal). The resulting number of digits to the left of the decimal point (or number of zeros to the right) is then found opposite this number (on the DN scale) on the RN-DZ scale. There are also Deci-Point scales for the trig and root functions. The use of Deci-Point scales on Pickett & Eckel slide rules was short-lived. Pickett used this system only on their first slide rule. The Pickett & Eckel Model No. 2 (Fig. 2), first issued in 1947, did not have the Deci-Point scales.

The Kamm Decimal-Keeping Scales. Lawrence Kamm patented [32] his "Decimal Keeping Scales" in 1959. These scales extend the range of the normal one-cycle C and D scales to 20 cycles from 1x10-10 to 1x10+10. Figure 6 shows the decimal-keeping scales on the back of the Pickett model No. N904-ES slide rule. On this slide rule there are matching decimal-keeping scales (K*, A*, B*, CI*, C*, D*, and L*) on the obverse for the normal K, A, B, CI, C, D, and L scales on the front. One makes the calculation with the decimal-keeping scales to get the proper order of magnitude (decimal point location) and the normal set of scales to determine the significant digits. For instance, for our example problem

No. 1, where we are determining the product of 462 x 0.00623, we find the answer to be [288] (with no decimal placement) with the C and D scales on the front of the slide rule. Now we turn the slide rule over, and make the same calculation with the C* and D* scales, and find the product to be about 2.9 x 10^0 or 2.9. Our answer to the problem is thus: 2.88. Note that we were able to obtain the result to approximately two significant digits with the decimal-keeping scale, but needed to use the normal scales to get the answer to three significant digits.

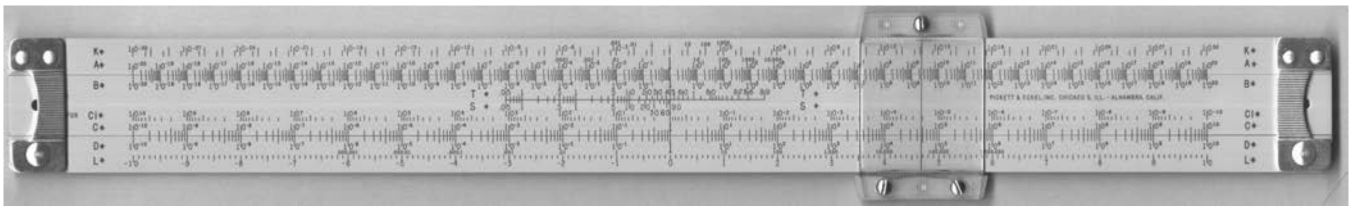


Fig. 6. The Kamm decimal-keeping scales.

Similar decimal-keeping scale sets are made for the A and B scales, and for the CI scale. Kamm's patent was used by Pickett on several of their slide rules, including model numbers 115, N901-T, and N904-ES. There is a curious similarity between the "decimal-keeping" scales (Fig. 6) invented by Kamm and the "Deci-Point" scales (Fig. 5) developed by Eckel 14 years earlier. The DN and RN-DZ scales on the Deci-Point slide rule can be thought of as a pair of 38-cycle scales, much like the 20-cycle set of scales on the "decimal-keeper" scales, the scale labels being thought of as the exponents (for the scientific method of expressing numbers). It is interesting that Eckel developed such a complicated scheme to keep track of the decimal point, and apparently did not appreciate the utility of the more direct approach of using just the multi-cycle pair of scales. Kamm [33] has claimed that after he assigned his patent to Pickett, they suppressed its use in their high-end engineering slide rules. There is a bit of intrigue here involving Pickett's use of decimal point-locating scales and devices. As we will see later, Pickett and Eckel had other patented devices to help locate the decimal point. To my knowledge, Pickett did not make any serious effort to make use of the decimal-locating technology available to them. Perhaps this should be the subject of a future article for the *Journal.*

It does not appear that any of these decimal-locating scale systems enjoyed much popularity. They were employed on slide rules that had relatively short production runs of only a few years, and all of the slide rules with these special scales are now fairly difficult for collectors to find. However, some special-purpose slide rules, such as Pickett electronic models N515-T and N16-ES, microwave transmission model N18-ES, and Air Force Aerial Photo model 520-T have the complete range of expected values over several decades for certain variables. These slide rules are fairly common.

Special Devices

Two types of devices have been used to help determine the location of the decimal point: the decimal-registering cursor, and the "decimalizer" slide rule.

Digit-Registering Cursors. I have identified three different digit-registering cursors that were available for slide rules, and one example of a homemade digit-registering cursor. All of these cursors fit Mannheim style slide rules - with scales on only one side.

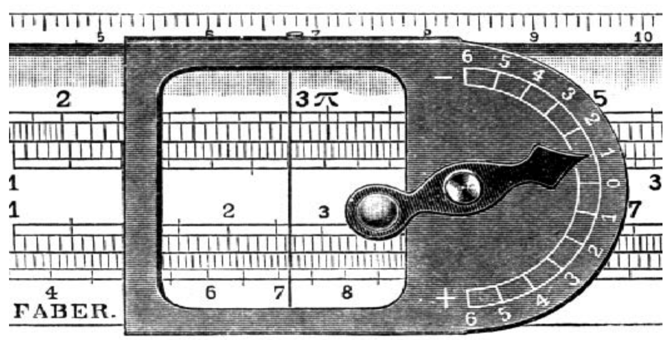


Fig. 7a. Faber.

Faber Registering Cursor. The earliest digit-registering cursor found was offered by Faber [16] for its Mannheim style slide rules in the early 1900s. The Faber "Registering Cursor" (Fig. 7a) has a semi-circular dial with pointer integral to the right side of the cursor window. The scale on the cursor ranges from -6 to +6. The pointer is used to keep track of the decimal point location during the calculation process. The settings are made by hand following each calculation in a problem. It looks like both the sums of digits (and zeros) and the order of magnitude shift can be kept track of with this cursor. However, because of its limited range, it would appear that the pointer is best used to keep track of the order of magnitude shift in a problem. Pickworth's [16] book shows this cursor on Faber models 367 and 380. I have a slightly later model 363 with this cursor. The Faber slide rules, on which the digit-registering cursor is found, commonly also have the cryptic markings as aids for making the digit count. Pickworth gave rules for the use of the registering cursor along with the cryptic markings.

Keuffel & Esser Decimal Pointer Indicator. Keuffel and Esser [34] show a metal-framed "Glass Indicator, with Decimal Pointer, model 4086" in their 1913 catalog. This decimal-point-registering cursor (Fig. 7b) looks much like the Faber, having a similar semicircular dial to the right of the cursor window. It also has a range of -6 to +6. It appears that this cursor is very rare; I have been unable to find a slide rule collector who has one.

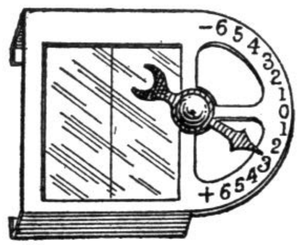


Fig. 7b. Keuffel & Esser.

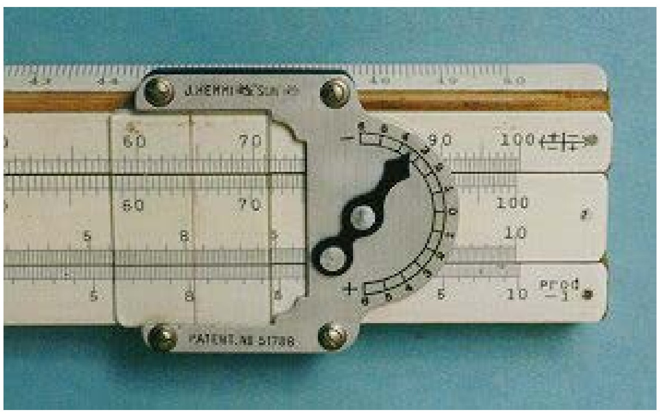


Fig. 7c. Hemmi.

Hemmi Digit Registering Cursor. Hemmi [35] patented a "Digit Registering Cursor" in Japan in the 1920s. Like the two others, the Hemmi cursor (Fig. 7c) has a semi-circular dial to the right of the cursor window. This cursor is the most attractive of the three cursors found, having a distinctive method of holding the glass so that its two vertical edges are not framed. It also has a range of -6 to +6. I have seen this cursor on two Hemmi slide rules, one with 10-inch scales and the other with 20-inch scales.

Given that K&E and Hemmi digit-registering cursors have similar configurations to the older Faber, and all have ranges from -6 to +6, it appears that they are copies of the original Faber invention. This seems even more likely for the Hemmi digit-registering cursor, as the slide rules on which it is found have the same cryptic markings as the early Faber slide rules with the Faber registering cursor. It does not appear that any of the digit-registering cursors enjoyed much popularity, as they are quite scarce, the K&E version being unknown in collections.

One other digit-registering cursor (Fig.7d) is in a private collection. This is a modification of a standard Dietzgen metal-framed cursor on a "Mack" slide rule. A series of beads slides along a wire fixed along the top edge of the cursor frame, the total count of beads being used for the digit count.

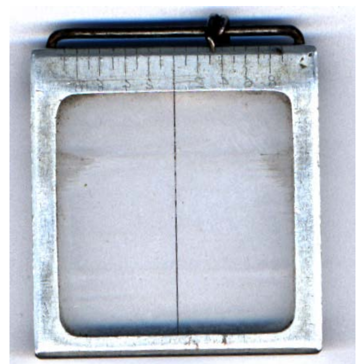


Fig. 7d. Modified Dietzgen.

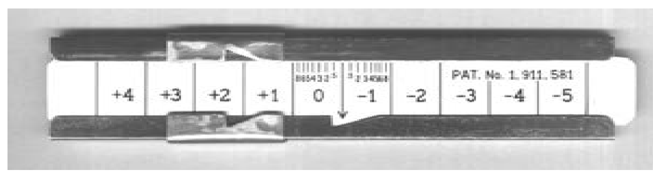
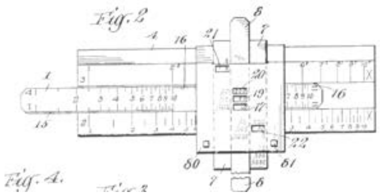


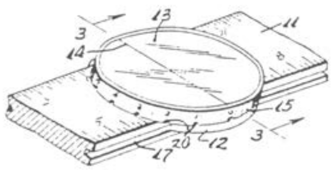
Fig. 8. Morse Decimalizer.

Morse Decimalizer. Morse's [36] decimal-point finder (Fig. 8) employs all of the standard elements of a linear slide rule: a body, a slide, and a cursor. It is, however, designed to be an accessory device to a normal slide rule. The slide has a linear scale divided into units from +4 to -5. The central units of 0 and -1 are subdivided according to log scale increments. The body and the cursor have index marks that relate to the scales on the slide. This is essentially a slide rule with a ten-cycle log scale. One must first reduce the problem to one of scientific notation, and make calculations using the cursor to maintain the position of the intermediate results. One can roughly estimate two significant digits, but additional significant digits must be obtained on a normal slide rule. Morse's decimal-keeping device was commercially produced in the 1940s.

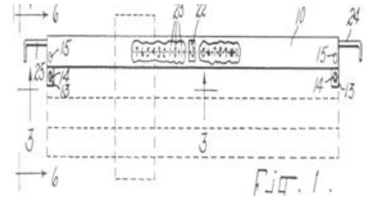
Figure 9. Decimal-Keeping Patents



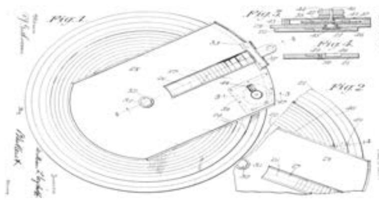
(9a) Smith (1912)



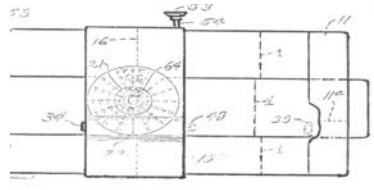
(9e) Cherney (1949)



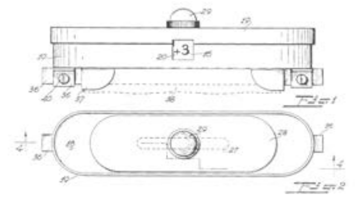
(9i) Smidl (1949)



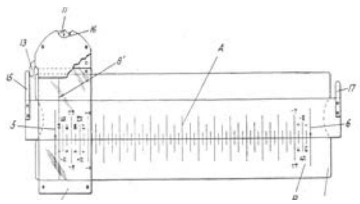
(9b) Wyckoff (1922)



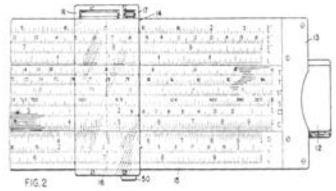
(9f) Eckel(1947a)



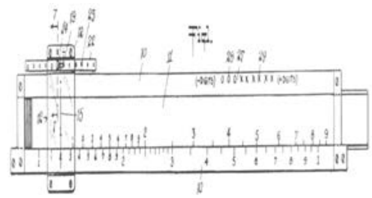
(9j) Smidl (1951)



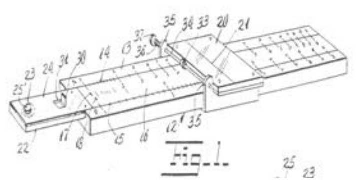
(9c) Schaeffer (1936)



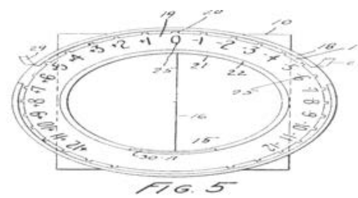
(9g) Eckel (1948)



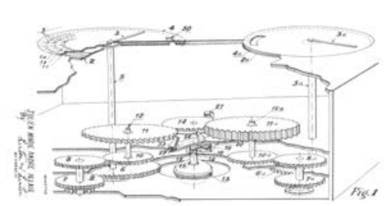
(9k) Christen (1950)



(9d) Cherney (1944)



(9h) Smidl (1947)



(9l) Allais (1953)

Numerous methods have been patented for keeping track of the decimal point in slide rule calculations. I have already discussed the special decimal-keeping methods patented by Eckel, Kamm, Wern *et al,* Hemmi, and Morse. Their ideas successfully made the leap from patent to marketable slide rule. But many others did not. Three distinct types of decimal-keeping mechanisms were identified. Most are mechanisms mounted on the cursor, or other parts of the slide rule. Others are stand-alone devices, and a few employ special scales.

Cursor or Slide Rule-Mounted Mechanisms

The challenge to develop a decimal-keeping mechanism for a slide rule started as early as the end of the 19th century, at about the time that the slide rule was gaining increased use. In 1897, Rudolph C. Smith [37] patented a special cursor that employed sliding parts and windows to keep track of the exponents during a calculation. Smith [38, 39, & 40] later refined this cursor to employ a transverse slide, and in 1912, he [41] made additional refinements, resulting in the cursor (and slide rule) configuration illustrated in Figure 9a. In the time since Smith developed his special cursor, there have been many more decimal-keeping cursors patented. Wyckoff's [42] cursor is shown in Figure 9b. His device is a digit-counting wheel mounted on the cursor of a circular slide rule. This device is an attempt at a device that automatically keeps track of the decimal point location. Each time the hairline on the cursor passes over the base line (index line) on the A scale, a detent on a wheel is engaged by a pin – causing the wheel to rotate one digit count.

George D. Schaeffer [43] patented a slide rule with a counting wheel mounted on the cursor (Fig. 9c). This slide rule is designed for adding fractions, the scales being laid out from 0 to 1 inch in 64ths of an inch. The digit-counting wheel is first set to the sum of the whole number parts of the numbers, and then the fractional parts are added. Each time the sum of fractional parts exceeds 1, a trip arm mounted on the left end of the slide engages a lever on the cursor, causing the rotation of the counting wheel one-unit of measure. While Schaeffer does not suggest an application of his device to slide rules with logarithmic scales, the same principle should hold. If the multiplication scales were for one cycle, such as for the C and D scales on an ordinary slide rule, then the counter wheel would keep track of the characteristic of the result. One would have to determine the sum of the characteristic, and set the counting wheel to this value. For the case where the denominator is greater than the numerator in an operation, the right end of the slide would have to be fitted with a trip arm, and the counting wheel fitted with a mechanism that allows its rotation one unit in the reverse direction.

Michael Cherney patented [44] a "Decimal Point Indicating Mechanism" (Fig. 9d) that attaches to the left end of the slide. When the cursor is moved to the index at the left end of the slide, the digit count is registered by a ratchet wheel, which is activated by a pawl on the cursor. The action is not totally automatic, as there is a button to push on the cursor at the appropriate time. Cherney later [45] received a patent on a much simpler digit-registering cursor (Fig. 9e) with a scale on the outer surface of a ring that frames the cursor window. There is an index pointer on the fixed part of the cursor to reference the scale on the ring to. One just turns the ring each time there is a change in the digit count.

Arthur F. Eckel [46] patented a "Calculator and Decimal Point Locator" (Fig. 9f) in 1947. This is a digit-counting device integral to the cursor on a slide rule. Two double disk dials are mounted to the back of the cursor to keep track of the digit count; one dial to keep track of the number of factors and the other the shifting of the digit count. A small lug on the left end of the slide engages the digit-counting mechanism when the left index on the slide is brought under the hairline on the cursor. The digit-counting mechanism automatically follows the same algorithm that was used on the Pickett and Eckel Deci-Point slide rule. In the next year (1948) Eckel [47] received another patent for a simpler digit-counting cursor (Fig. 9g) mechanism. Finally, in 1949 Eckel [30] obtained a patent for a digit-registering cursor that automatically keeps track of the digit count. This cursor has a gear that rides on a gear rack fastened to the slide. Eckel's 1947 and 1949 patents also describe circular slide rules with digit-registering dials.

Joseph Smidl patented three different decimal-registering mechanisms for slide rule cursors. His 1947 patent [48] shows a device (Fig. 9h) that consists of a circular ring - with a digit counting scale - mounted on the cursor so that the window is visible through the inside of the ring. The ring is rotated by hand to record changes in the digit count, the total count being visible in a small window. Later (1949) Smidl [49] obtained a patent for a digit-registering device (Fig. 9i) that attaches to the upper stator on a slide rule. The device consists of a linear bar that slides inside of a metal housing. Levers at each end of the track activate the digit-counting bar one unit for each push. The digit count is viewed through a small window. In 1951, Smidl [50] patented a more complex device (Fig. 9j) that slides in the same grooves as the cursor. It can be pushed along in concert with the cursor, or one edge can be used as the reference line in place of a cursor with a window and hairline. This device consists of a link belt that rotates in a continuous track-way. Each link of the belt is marked with a digit count, the count being displayed in a small window on one side of the cursor. A double-acting button in a slot on the top of the device is used to increase or decrease the digit count.

In 1950, *Charles Christen* [51] patented a "Decimal Point Locator for Slide Rule" that automatically keeps track of the decimal point during slide rule operations. Christen's device (Fig. 9k) includes a sliding tally bar, integral to the cursor, a pin that fixes the tally bar in position with the cursor according to the number of digits in the multiplicand and multiplier, and a reference scale adjacent to the tally bar on the upper stator. The tally (in concert with the cursor) bar automatically determines the decimal point location when a decimal point on the cursor is positioned over the reference scale. Rather than registering a digit count number, this device places an index pointer along a scale having only the labels "0 0 0 X X X X X X".

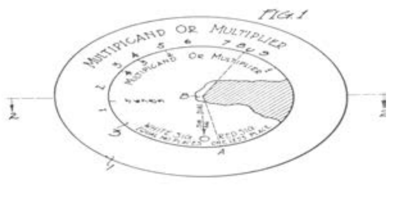
The last, and most complex, of the decimal point-registering devices that I found was patented by *Julien Marie Andre Allais* [52] in 1953. Allais's device (Fig. 9l) consists of two circular slide rule dials connected by a gear and shaft mechanism. Calculations are done on the primary dial, and the decimal-point tracking automatically follows on the accessory digit accumulator dial. A complex of gears and shafts makes up the works of this device.

As far as I know, the only one of these patents that was incorporated into a commercially successful slide rule was the digit-registering cursor by Jirou Hemmi.

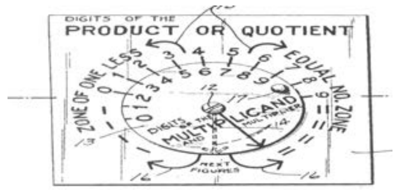
Stand-Alone Decimal Locating Devices

Charles E. Latshaw [53] patented a "Decimal Dial" device (Fig. 10a) for locating the decimal point in arithmetical operations. Its configuration is that of a two-disk circular slide rule. The outer and inner disks have scales ranging from 1 to 9, the outer scale running clockwise, the inner scale running counterclockwise, the index for 5, 6, 7, 8, and 9 being the same point. The spacing of the scale numbers is not uniform but is "peculiar", according to Latshaw. There is a small "aperture" in the inner disk, and a red sector on the outer disk. The remainder of the outer disk is white. The number of digits resulting in the multiplication of two numbers is determined by lining up the leftmost digits in the multiplier and multiplicand. If the color showing through the aperture is white then the number of digits in the product is equal to the sum of the number of whole numbers in the two numbers. If a red color shows through the aperture, the number of digits in the result is the total number of digits minus one. Other rules apply to multiplying numbers smaller than zero, and division operations.

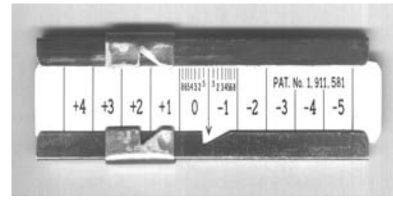
Figure 10. Digit-registering devices on slide rules.



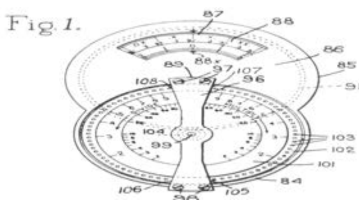
(10a) Latshaw (1919)



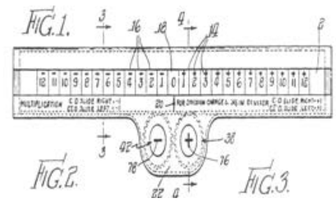
(10b) Latshaw (1923)



(10c) Morse (1933)



(10d) Gilmore (1938)



(10e) Dickson (1943)

Later, in 1923, *Latshaw* [54] patented a "Decimal Point Calculator". This device (Fig. 10b) is an improvement on Latshaw's earlier patent. Its configuration is that of a two-disk circular slide rule. The outer and inner disks have scales ranging clockwise from 1 to 9 in the upper segments. The spacing of the scale numbers is uniform. There are minus signs around the lower left quadrant and equal signs around the lower right quadrant on the fixed disk. There is an arrow on the rotating disk pointing to its lower edge. The number of digits resulting in the multiplication of two numbers is determined by lining up the leftmost digit in the multiplicand on the inner rotating disk with the leftmost digit of the multiplier on the outer fixed disk. If the arrow points at an equal sign, then the number of digits in the product is equal to the sum of the number of whole number digits in the two numbers. If the arrow points at a minus sign, the number of digits in the result is the total number of digits minus one. Other rules apply to multiplying numbers smaller than zero, and division operations.

In 1933, *George Morse* [55] patented a "Decimal Point Finder" for "quickly locating the decimal point when making computations". This device (Fig. 10c) has a rotating hard rubber disk contained between two plates. The upper plate reveals three pie-shaped segments of the disk. The disk is marked in ten segments from -5 to +5, one half of each segment also being marked with a plus sign followed by a digit, the other half marked with a minus sign followed by a digit. A radial line separates the two halves of each segment. There is also a fixed index pointer mounted to the base of the device. Both edges of each pie-shaped window in the upper plate are marked radially, the opening adjacent to the pointer with 7, 8, 9, and 1, the opening 90 degrees counterclockwise with the numbers 4, 5, 6, and 7, and the opening opposite the pointer is marked with the numbers 1, 2, 3, and 4. One needs to first sum the characteristics of each factor in the operation to determine the resulting characteristic. The disk is then rotated so that the index pointer is lined up with the corresponding radial line number. Each factor is then picked up in the appropriate window and along the appropriate edge (watching for the sign of the number). A sharp pointer is used to mark the point on the disk and rotate the disk until the needle point contacts the opposite edge of the window. For division the numbers are considered negative, and the disk is rotated in the other direction. The resulting characteristic will be found in the sector opposite the index pointer.

Later (1938) Morse [36] patented a Decimilizer device for accurately locating the decimal point when making computations. As can be seen from the previous discussion of this device (Fig. 8), it is a radical departure from Morse's previous patent. It is more like a linear slide rule, and simpler in construction and operation. Morse's decimal-registering device was commercially produced in the 1940s. Models of two different lengths (5-inch and 10-inch) are known.

In 1938, *Howard Gilmore* [56] patented a "Decimal Point Indicating Mechanism for Slide Rule Computations." Gilmore's device (Fig. 10d) resembles two overlapping circular disks. The lower disk has two single-cycle log scales, one on the right half for multiplication and the other on the left half for division. A vertical, narrow, bow-tie-shaped plate separates the two scales and provides stops for the beginning and end of each scale. Detents about the perimeter of the disk facilitate its rotation with a stylus. The upper disk rotates behind a window. It has a scale that shows the characteristic of the number. It is connected through a gear system to the lower disk. The calculation is performed by placing the stylus in the detent closest to the first number in the operation, and rotating the lower disk with the stylus until the stylus makes contact with a stop. The gear system is then disengaged and the lower disk is returned to its neutral position, wherein the gear system is re-engaged. The stylus is then placed in the detent closest to the second number in the operation and the dial is rotated again to its stop. The procedure is repeated until all operations in the calculation are completed. The characteristic (or digit count) of the result will be shown in the window in the upper part of the device.

In 1943, *Charles Dickson* [57] patented a "Decimal Finder" device. Dickson's decimal finder can be made in a linear (Fig. 10e) or circular form. It employs a slide that is divided in linear units from -12 to +12. Two knobs turn gears that engage a linear gear along the length of the slide. An index mark references the number scales on the slide to the body of the device. The mechanics are that when the push button marked with a plus sign is pushed, the slide is advanced one unit, and when the push button marked with a negative sign is pushed, the slide is reversed one unit. To use the decimal finder, the slide is initially set so that the zero position is opposite the index mark. The operation of the decimal finder is then made in concert with the operation of the C and D scales on the slide rule. For the first two numbers in a multiplication operation, the characteristics are added and the positive or negative button is pushed the appropriate number of times to advance the decimal finder slide. The common rules for digit counting and slide extension apply. The characteristic for the result of a calculation will appear on the slide (on the decimal finder) opposite the index pointer.

Of the above reviewed stand-alone devices, only the second of Morse's patents is known to have been commercially developed.

Special Decimal-Locating Scales. It appears that three of the decimal point-locating scale systems previously discussed were patented: the "Deci-Point" system by Arthur Eckel [30] in about 1949, the Lawrence Kamm [32] "Decimal Keeping" system in 1959, and the Wern family [26] "System Wern" in 1968. All of these patents have been incorporated into slide rules, but none of the slide rules appears to have sold in significant quantities. Few have survived to make it into collections.

Last Remarks

Many attempts were made in the past to develop decimal point-locating methods and devices. Some of the methods are straightforward, some overly complicated. Many of the devices are ingenious, and others extremely complex. There was a recognition on the part of some slide rule practitioners that there was a need for specific rules or for devices to help in the locating of the decimal point. Yet few of their efforts received widespread acceptance and recognition. Pickett & Eckel seems to have had the greatest interest in providing rigorous means for determining the decimal point position. They incorporated two special scale methods into their slide rules, and obtained several patents for decimal-registering cursors. The most important of the U.S. slide rule manufacturers for all of the first half of the 20th century, Keuffel & Esser, appears to have had little interest in this problem. Other than the Faber digit "Registering Cursor", and the "System Wern" direct-reading scales, I found little evidence that slide rule makers in Europe were interested in providing help for the slide rule user in locating the decimal point.

Special devices for determining or registering the decimal point are rare. The most common seem to be the early Faber decimal-registering cursor and the Pickett & Eckel "Deci-Point" and "Decimal Keeper" slide rules. Even these are fairly scarce. While there was considerable activity in developing special decimal-locating devices and methods in the U.S., there was little interest overall in using special devices or scales for keeping track of the decimal point. Most slide rule users appear to have been content to use the mental inspection and approximation methods, reverting to some form of digit counting method for the more complicated problems.

Acknowledgements

Many slide rule collectors have volunteered information for this paper. Special thanks are in order to Bobby Feazel for the image of the early Dietzgen modified decimal-registering cursor, Conrad Schure for the image of the System Wern slide rule, Rodger Shepherd for the abundance of decimal-keeping literature, Lawrence Kamm for permission to quote from his history of his decimal-keeping scales, Wayne Feely for his timely offering of two Pickworth's books, Don Digalblo for his offering of the eclectic comprehensive rules by Clyde Clason, Dick Rose for reminding me of the K&E 4086 decimal-registering cursor, Bob Otnes for the Hoare slide rule, Andrew Davie and Rick Blankenhorn for providing forums for posting requests for information, Larry Blackwelder, Walter Shawlee, and Steve Kays for helping to identifying all of the Pickett slide rules with the decimal-keeping scales, Al Mosby for reminding me of the Faber decimal-registering cursor, and finally Bob DeCesaris for determining that there are no decimal-registering devices in his extensive collection of B.K. Elliot, Kolesch & Co, Hughes-Owens, Bruning, Dietzgen, and Leitz catalogs. I should also like to express my appreciation to the patient librarians at the State Library in Concord, New Hampshire for providing me access to the U.S. Patent Depository, and hauling the hundreds of volumes of dusty *Patent Gazettes* from storage for me to read. Finally, I need to express my gratitude to my wife, Ingrid, for her patient explanation to our friends and relatives of my preoccupation with the decimal point.

References

1. Cajori, Florian, *William Oughtred - A Great Seventeenth Century Teacher of Mathematics,* The Open Court Publishing Co., Chicago & London, 1916, 100p.

2. Pickworth, Charles N., *The Slide Rule,* 12th ed., Edinburg Colstons Limited Printers, 1910.

3. Breckenridge, *The Polyphase Slide Rule,* Keuffel & Esser Co., New York, 1924.

4. Hartung, Maurice L., *How to use the Deci-Log Log Slide Rule - Model No.3,* Pickett & Eckel, Inc., Chicago, 1947.

5. Post, Frederick, *Post Versatrig Slide Rule Instructions,* Frederick Post Co., Chicago, 1968.

6. Thacher, Edwin, *Directions for Using Thacher's Calculating Instrument,* Keuffel & Esser Co., New York, 1920, 71p.

7. Culimore, Allan R., *The Use of the Slide Rule,* Keuffel & Esser Co., New York, 1920.

8. Hills, E. Justin, *A Course in the Slide Rule and Logarithms,* Ginn and Co., 1943.

9. Johnson, Lee H., *The Slide Rule,* D. Van Nostran Co., New York, 1949.

10. Kells, L.M., Kern, W.F. & Bland, J.R., *K&E Slide Rules – A Self Instruction Manual,* Keuffel & Esser Co., New York, 1955.

11. Stanley, W. & Co., *Stanley's Slide Rule Instructions,* Stanley, W. & Co, Peru, Indiana, 1939.

12. Asimov, Isaac, *An Easy Introduction to the Slide Rule,* Houghton Mifflin Co., Boston, 1965.

13. Cox, W.M., *The Mannheim Slide Rule - Complete Manual,* Keuffel & Esser Co., 127 Fulton Street, New York, ca. 1900, 32p.

14. Poland, John, *The Quick and Easy Slide Rule Instruction Book,* Engineering Instruments Co., Peru, Indiana, ca. 1930s, 28p.

15. Bishop, Calvin C., *Slide Rule – A Practical Guide to Its Use,* 3rd Ed., Barns & Noble, New York, 1955.

16. Pickworth, Charles N., *Instructions for the Use of A. W. Faber's Improved Calculating Rule,* A. W. Faber, Newark, N.J., ca. 1904, 53p. + 14 fold-out sheets.

17. Richardson, G.W & J.J. Clark, *The Slide Rule Simplified,* Sixth Edition, 1918, 100p.

18. Slater, A.F., *The Slide Rule – A Complete Manual,* Holt, Rinehart & Winston, 1967, 291p.

19. Stanley, W.F. & Co. Ltd, *Instructions for the Use of the Fuller Calculator,* London, ca. 1920, 28p.

20. Zanotti, George J., *Post Versalog II Slide Rule Instruction Manual and Applied Text,* Teledyne Post, Chicago, 1970.

21. Hartung, Maurice L., *How to Use Trig Slide Rules,* Pickett Slide Rule Co., 1960, 64p.

22. Ellis, John P., *The Theory and Operation of the Slide Rule,* Dover Publications, New York, 1961.

23. Hemmi Slide Rule Co., *Instruction Manual for Hemmi 269 Slide Rule,* Hemmi Slide Rule Co., Tokoyo, ca. 1970.

24. Clason, Clyde B., *Delights of the Slide Rule,* Thomas Y. Crowell Co., 1964.

25. Hoare, Charles, *The Slide Rule and How to Use It,* London, 1867.

26. Wern, C.R., Wern, G.H., and Wern, L.A., Slide rule, U.S. Patent No. 3,374,947, Mar. 26, 1968.

27. Schure, Conrad, "The System Wern Slide Rule", Journal of the Oughtred Society, v.6, n.2, 1997.

28. Riehle, Harald, "The Development of Data Slides from 1920 Until Today", *Proceedings* of Third International Meeting of Slide Rule Collectors, Faber-Castell Castle, Stein/Nurnberg, Germany, 1997, p. 23-31.

29. Eckel, A.F. Deci-Point Slide Rule. Copyright 1945.

30. Eckel, A.F., Calculator and Decimal Point Locator, U.S. Patent No. 2,466,883, April 12, 1949.

31. Hartung, Maurice L., *How to use the Deci-Point Decimal Point Locator and Slide Rule,* Pickett & Eckel, Chicago, 24p.

32. Kamm, Lawrence J., Decimal Keeper Slide Rule, U.S. Patent 2,893,630, 1959.

33. Kamm, Lawrence J., "Adventures of an Entrepreneur or War Stories from American Industry", Kamm Internet Home Page, (http://members.aol.com/lkamm18392/adventur.htm), 1987.

34. Keuffel & Esser, Catalog of Keuffel & Esser, New York, p. 305, part no. 4086, 1913.

35. Hemmi, Jirou, Slide-Rule, Japanese Patent No. 51788.

36. Morse, George H., Slide Rule, U.S. Patent No. 2,158,476, May 16, 1938.

37. Smith, Rudolph C., Calculating-Scale, U.S. Patent No. 592,067, Oct. 19, 1897.

38. Smith, Rudolph C., Calculating-Scale, U.S. Patent No. 746,888, Dec. 15, 1903.

39. Smith, Rudolph C., Calculating-Scale, U.S. Patent No. 746,889, Oct. 19, 1897.

40. Smith, Rudolph C., Calculating-Scale, U.S. Patent No. 920,497, May 4, 1909.

41. Smith, Rudolph C., Calculating-Scale, U.S. Patent No. 1,014,344, Jan. 9, 1912.

42. Wyckoff, W.L., Calculating Rule, in U.S. Patent No. 1,423,896, July 25, 1922.

43. Schaeffer, George D., Slide Rule, U.S. Patent No. 2,061,853, November 24, 1936.

44. Cherney, Michael, Decimal Point Indicating Mechanism for Slide Rule Computations, U.S. Patent No. 2,363,642, Nov. 28, 1944.

45. Cherney, Michael, Digit Indicator for Slide Rules, U.S. Patent No. 2,468,812, May 3, 1949.

46. Eckel, A.F., Decimal Point Locator for Slide Rules, U.S. Patent No. 2,426,601, 1947.

47. Eckel, A.F., Decimal Point Locator for Slide Rules, U.S. Patent No. 2,446,696, Aug. 10, 1948.

48. Smidl, J.J., Slide Rule Cursor, U.S. Patent No. 2,418,910, April 15, 1947.

49. Smidl, J.J., Slide Rule Attachment, U.S. Patent No. 2,490,240, Dec. 6, 1949.

50. Smidl, J.J., Characteristic Accumulator for Slide Rule Cursors, U.S. Patent No. 2,579,019, Dec. 18, 1951.

51. Christen, Charles, Decimal Point Locator for Slide Rules, Oct. 24, 1950.

52. Allais, J.M.A., Disk Slide Rule with Decimal Point Indicator, U.S. Patent No. 2,658,671, Nov. 10, 1953.

53. Latshaw, Charles E., Decimal-Dial, U.S. Patent No. 1,301,140, April 22, 1919.

54. Latshaw, Charles E., Decimal-Point Calculator, U.S. Patent No. 1,474,533, Nov. 20, 1923.

55. Morse, George H., Decimal Point Finder, U.S. Patent No. 1,911,581, May 30, 1933.

56. Gilmore, Howard, Decimal Point Indicating Mechanism for Slide Rule Computations, U.S. Patent No. 2,117,413, May 17, 1938.

57. Dickson, Charles L., Decimal Finder, U.S. Patent No. 2,328,966, Sept. 7, 1943.