Some Notes on the History and Use of Gunter's Scale

If you are interested in the history of slide rules, chances are that, at some time in your life, you had some instruction, either formal or informal, on their use. But what about your knowledge of the use of the Gunter scale? Have you ever tried to solve a mathematical problem with a pair of dividers and just one logarithmic scale?

The Gunter scale was invented by Edmund Gunter in 1620 and was the link between Napier's logarithms, which were invented (or should I say discovered?) in 1614, and Oughtred's slide rule which was invented around 1630. Gunter created the scale by laying out a table of logarithms on a rule using marks to designate the values of the logarithms. On this rule, the locations of the marks were proportional to the values of the logarithms. The end result was almost identical to the A or B scales on modern slide rules and was referred to as Gunter's line of numbers, and was usually designated on the rule as "NUM".

In addition to the line of numbers, Gunter scales eventually evolved to include at least seventeen other scales for use in trigonometry and navigation. Cajori (1920) quotes a description of a rather simple Gunter scale from 1624 as having, in addition to the line of numbers, a line of tangents, a line of sines, a line one foot in length, divided into 12 inches and tenths of inches, and a line one foot in length divided into tenths and hundredths. Cajori (1909) includes a copy of a drawing of a slightly more elaborate Gunter scale from M. Bion's book of 1723, showing the same lines, arranged in the same order, as is shown in the lower portion of Figure 4.1. In a mathematical text printed in 1799, Alexander Ewing described a nearly identical Gunter scale consisting of the same eight lines as shown in Bion's illustration. Both Bion's illustration and Ewing's text includes four additional scales not found on the 1624 rule. These are: S. R. (sines of the rhumbs), T. R. (tangent of the rhumbs), Mer. (meridian), and E. P. (equal parts) which, according to Ewing (1799), are used only for navigation.

The above authors make no reference to any lines on the reverse side of these rules. The Gunter scale owned by the author, which was made by J. D. Potter sometime between 1830 and 1861 (Taylor, 1966), (a drawing of which is shown in figure I) contains 18 scales of various types with scales appearing on both sides of the rule. This arrangement appears to be nearly identical to a late, 17th-century Gunter scale in the Science Museum that is described by Baxandall and Pugh. Jeremiah Day, the President of Yale College, in the third edition of his book *The Principles of Plane Trigonometry, Mensuration, Navigation and Surveying* written in 1831, indicates that two different names were used for the two sides of the Gunter scale. He states:

> "To facilitate the construction of geometrical figures, a number of graduated lines are put upon the common two feet scale; one side of

which is called the Plane Scale, and the other side, Gunter's Scale."

Gunter scales were usually made on strips of boxwood 24 inches long, approximately 1 $\frac{3}{4}$ inches wide and roughly $\frac{1}{4}$ inch thick. Many of them had small brass pins inserted into the wood at points of heavy use to protect the wood from damage by the points of the dividers that were used to measure off values on the scales. These pins are shown as solid black dots in Figure 4.1. There were also variations in size and materials. Baxandall and Pugh list a brass Gunter scale by Andrew Yeats as being in the Science Museum in London. Belcher Brothers and Company of New York offered both one- and two-foot Gunter scales of both boxwood and satinwood in their 1860 catalog. The two-foot scales were available unbound, half-bound and bound. According to Turner's *Antique Scientific Instruments,* Gunter's line of numbers (or NUM scale) also appeared on some sectors, and Kebabian (1991) describes a two-foot, two-fold carpenter's rule with Gunter's line of numbers inscribed on one leg, obviously a precursor to the Gunter slide that later became so common on these rules.

After Gunter created his rule, all that remained for Oughtred to do to invent the slide rule was to lay two of Gunter's lines of numbers side by side, as A and B scales. Having done this, he created a device that would be indispensable to businessmen, scientists, astronomers, navigators, engineers and others for nearly 350 years. In fact, the two-foot, two-fold wooden, brass or ivory carpenter's and engineer's rules, with slide rules built into them, were known throughout the 19th century as Gunter rules, even though they were truly slide rules.

Instructions on the use of the Gunter scale were included in early books on arithmetic and navigation. The *Encyclopaedia Britannica,* in its 1771 edition, dedicated nearly a full page to instruction and descriptions relating to Gunter's line and Gunter's scale. Two examples of these texts are mentioned by Cajori (1909) in his book on the history of the logarithmic slide rule. One of these texts is Bowdich's *Navigator,* and the other is Nicholas Pike's *A New and Complete System of Arithmetic.* Pike's book, which was published by the colonial printer Isaiah Thomas in 1788, is the earliest book printed in America that is mentioned by Cajori as including instructions on the use of the slide rule. Not having seen the first edition of this book, the author recently acquired a copy of the second edition which was published in 1797, and was surprised to find that this edition contained no description of the slide rule and no detailed instructions on the principles and use of either the slide rule or Gunter's scale. Instead, Pike seems to have assumed that the reader had a working knowledge of both of these devices, and proceeded merely to give the settings required to solve a variety of problems using both Gunter's scale and the sliding rule. It seems very interesting that this book, published before 1800, starts out with the basic rules of addition and subtraction, and covers trigonometry, geometry, currency conversions, compound interest, present value, etc., and yet assumes that the reader has a working knowledge of both the slide rule and the Gunter scale.

What is also surprising about this book is simply the fact that, along with the settings for the solutions of various problems by the use of the "sliding rule", are the descriptions of the manipulations of the dividers required to solve the same problems using Gunter's scale. The references to the sliding rule and to the Gunter are to be found only in the section of the book on the measurement of surfaces and solids. This is the section of the book that also includes references to the practice of gauging.

Before proceeding to Pike's use of the Gunter scale in the solutions to some common problems, it may be worthwhile to cover some of the basic operations using the line of numbers on the Gunter scale. Multiplication is accomplished by measuring the distance from the left index to the first number, and then adding this to the distance from the left index to the second number. Thus to multiply 12 times 5, you would set the dividers to span the distance from the left index to 12 and them move them to where the left leg is on 5 and the result, 60, will be found under the right leg. Or, in other words, the sum of the distances from the left index to each of the numbers to be multiplied is equal to the distance from the left index to the product. (This is true because the sum of the logarithms is equal to the log of the product.) Division, likewise, is accomplished by merely measuring the distance of the denominator from the left index, and subtracting it from the distance from the left index to the numerator. Thus, to divide 18 by 3, you would merely set the dividers to the distance from the left index to 3 and then move the dividers to where the right leg is on 18 and the answer, 6, will be found under the left leg. Squaring is very simple; it is accomplished by merely flipping the dividers once over. The square root is slightly more complicated in that it is necessary to set the dividers to one half of the distance from the left index to the number of which the root is to be taken. Cube roots would be one third, fifth roots one fifth, etc. It is the author's guess that the E. P. (equal parts) scale could have been used in the calculation of powers and roots. For example, to calculate the cube root of 27, first set the dividers to span from the left index to 27. Then measure the span of the dividers on the equal parts scale. In this case it would be 132 units. Divide this by three (to get the cube) and get 44 units. Now set the dividers to 44 on the E. P. scale and transfer this to the line of numbers (NUM scale) and with the left leg of the dividers on 1, the right leg will be on 3, the cube root of 27. A set of proportional dividers was sometimes used to assist in calculating both roots and powers.

Nicholas Pike's book of 1797 includes many examples of solutions of problems through the use of first the sliding rule and then the Gunter scale. Two of these, which were selected because they are rather simple, have been included here. These problems illustrate not only the use of the sliding rule and the Gunter, but they also include some simplistic insights into 18th century arithmetical methods. For an example, Pike did not use the units of "square inches" or "square feet" in these problems nor did he use π in the problem involving the area of a circle. Also, his harsh economy of words in describing the solutions seems to be typical of that era.

The first example is taken from page 427 and simply involves the calculation of the area of a square. "Rule. - Multiply the side of the square into itself, and the product will be the area of superficial content, of the fame name with the denomination taken, either in inches, feet, or yards, respectively. Let ABCD represent a square, whose side is 12 feet. Multiply the side 12 by itself, thus,

$$\begin{array}{rrr} & 12\text{ inches.} & 12\text{ feet.} \\ & \underline{12\text{ inches.}} & \underline{12\text{ feet.}} \\ \text{Area} = & 144\text{ inches.} & 144\text{ feet.} \end{array}$$

By the Sliding Rule

Set 1 to the length on B, then, find the breath on A, and opposite to this on B, you will have the content.

By Gunter's Scale

Extend the dividers from 1, on the line of numbers, to the length; that distance, laid the same way from the breadth, will point out the answer."

A second example, from page 438, involves calculating the area of a circle when the diameter is known.

"The Diameter being given, find the Area of a Circle without finding the Circumference.

Rule - Multiply the square of the diameter by .7854, and the product will be the area of the circle, whose diameter was given.

Example. The diameter of a circle being 12, to find the area?

$$\begin{array}{rl} & .7854 \\ 12\times 12 = & \underline{144} \\ & 3\,1416 \\ & 31\,416 \\ & \underline{78\,54} \\ & 113.0976 = \text{area} \end{array}$$

By the Sliding Rule

Set 1 on A to the diameter on B, then find .7854 (which expresses the area of the circle whose diameter is 1) on A, against which on B is a 4th number, then find this 4th number on A, against which on B is the area.

By Gunter

The extent from 1 to the length of the diameter reaches from .7854 to a 4th number, and from that 4th number to the area."

An excellent, and somewhat more recent, description of the use of the Gunter scale can be found in Ken Roberts' recent reprint of Rabone's *The Carpenter's Slide Rule, Its History and Use,* which was originally printed in 1880.

By using a pair of dividers along with the B scale on the slide out of almost any slide rule, the reader can quickly confirm two things. First, that yes, calculations can be made in this manner, and secondly, that Oughtred had good reason to try to find a more convenient way to use Gunter's scale.

The question, however, that seems to beg for an answer is: why was there still a market for Gunter scales more than 250 years after the slide rule was invented? It would seem that the slide rule, with its greater ease of use, would have replaced Gunter's scales some time prior to 1700. Oddly enough, some were still being sold near the end of the 19th century, and some arithmetic, surveying and gauging texts still contained descriptions of their construction and instructions on their use well into the 19th century. For example, Davies' *Elements of Surveying and Navigation,* in both the 1853 and 1868 editions, contains two paragraphs on the use of Gunter's scale. One paragraph mentions only the scale of equal parts, the diagonal scale of equal parts, and the scale of chords. The other paragraph is dedicated to the use of the line of meridional parts. Oren Root's *New Treatise on Surveying and Navigation* of 1867 allows a page and a half for discussion of the use of Gunter's scale, but prefaces it with the following comment: "Gunter's scale is commonly two feet in length, containing the plane scale, and the scale of sines, chords, and tangents on one side of it, and the scale for logarithms of numbers, sines and tangents on the other. The logarithmic scale is not much used." John Rabone and Sons, rule makers in Birmingham, England, listed a 24" Gunter's navigation scale in their catalog as late as 1892. However, not all rule makers offered Gunter scales at this time. Even though the Stanley Rule and Level Company was a prolific manufacturer of rules in the second half of the 19th century, it does not appear that they offered any Gunter scales. They did, however, offer folding engineer's and carpenter's rules with Gunter slides (See *The Stanley Catalog Collection* 1989).

It would seem that for Gunter scales to be offered for sale as late as 1892, they must have had some advantage over slide rules. One advantage may have been the result of a combination of price and accuracy. In the 1860 Belcher Brothers & Co. catalog, two-foot, two-fold carpenter's rules with Gunter slides were \$10.50 to \$12.50 per dozen (88 cents to \$1.04 each), depending on the type of joint chosen. In the same catalog, two-foot Gunter scales were \$10.00 per dozen (83 cents each). In Rabone's 1892 catalog, both of these rules were \$20.00 per dozen (\$1.66 each). Keuffel and Esser, in their catalog dated the following year, priced their 10-inch Gunter slide rule at \$3.50 each and their 10-inch and 20-inch Mannheim rules at \$4.50 and \$10.50 each, respectively.

A second advantage may have been greater accuracy. It would seem that the Gunter scale, being twice as long as the slide on the carpenter's rule, could have been at least twice as accurate.[3]

[3]The carpenter's rule also has two cycles on the scales commonly used for multiplication. *Ed.*

A third advantage of the Gunter scale may have been the simple fact that the use of the Gunter scale was so ingrained in the habits of those who used them that they merely resisted the change to the slide rule. Williams (1985) in his book *A History of Computing Technology* makes a comment that supports this idea quite clearly:

> *"Indeed the early examples of slide rules which still survive usually show unmistakable signs of having been used, not in the intended way, but by having had a pair of dividers pick off lengths along the logarithmic scales."*

It may be interesting to note that a book published by Dietzgen as late as 1905 (Rosenthal, 1905) includes instructions on how to use a pair of proportional dividers to determine powers on a slide rule.

A fourth possible advantage of the Gunter scale over the slide rule may have been that, because it had no moving parts, it was not subject to sticking and binding in humid nautical settings. Its use in navigation is confirmed by Joseph Bateman, an expert on the slide rule (Taylor, 1966) who observed about 1840 that navigators still used a Gunter scale with compasses. As mentioned above, the Gunter scale listed in the 1892 catalog was described as a "Gunter's Navigation Scale."

And one last advantage may have been simply that with the large number of scales, the Gunter scale may simply have filled a need that was not filled with the relatively few scales found on slide rules. Attempting to compare a slide rule with a Gunter scale may be analogous to comparing an adding machine to a book of mathematical tables.

It appears that through the 18th and 19th centuries the Gunter scale may have been used more for general trigonometric calculations and navigational calculations and that the slide rule was used more for specific technical applications. Many slide rules made prior to 1880s have special scales and gauge points for timber, tax assessment, pumping engine design, pipe sizing, etc.

The author does not know how common Gunter scales are today, and would be interested in any information that might be available from other scholars or collectors.

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