

Aspect Ratios of Pictures and Their Calculations by Slide Rule

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Introduction

The functions of the human eye are delimited by the angle of view. The viewing angle of an eye is not a sharp circle but a pluriform frame defined around view angles of some 60° upwards, 75° downwards, 60° towards the nose, and even 95° away from the nose. Within that viewing frame the quality of vision is not evenly distributed, but best in the area we focus on, diminishing towards the periphery, and of course worst in the “blind spot”.

When we use one eye, while covering the other, we see an egg-shaped field, lying with the top outwards and slightly cut out by the nose. However, when we look through both eyes, the field of view looks like a horizontal band that is very much wider than high.

Although we look at the real world within our own eyes’ field of vision, technology has given us the means to look at artificial pictures - images of reality, of design plans, of art, either static or moving. Not surprisingly, the frames of pictures in general mimic the field of view of the eye in a simplified form: the rectangle in a “lying” format (width greater than height). Standing formats (height greater than width) also exist in picture framings - though not matching nicely the eyes’ field of view. For example a person standing, the portrait of a face, the architecture of a tower, etc. Photographers call the lying format “landscape”, and the standing format “portrait”.

This article will describe a selection of frame aspect ratios in various kinds of images. Calculations for aspect ratios are described, followed by an easy method using the slide rule.

Picture Frames in Imaging Technology

During the last two centuries, three imaging technologies have made their varying choices of aspect ratios: photography, movies, and TV or PC screen. Table 1 shows the main aspect ratios in picture technologies of past and present. The entries in the table are arranged in order of increasing width:height ratio.

Sizes of TV and PC screens are occasionally specified not by height “a” and width “b”, but by height-to-width ratio **a:b** and the length of the screen’s diagonal **d**, see Figure 1.

The Pythagorean relation between diagonal, height and width is:

$$d = \sqrt{a^2 + b^2} = b\sqrt{a^2/b^2 + 1} = b \cdot K \quad (1)$$

with $K = \sqrt{a^2/b^2 + 1}$ only depending on ratio a:b.

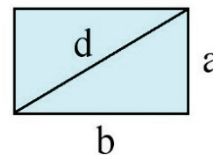


FIGURE 1. Frame Dimensions

The square is the most basic format – no choice needed between lying and standing – and easy for the eye to focus and concentrate on. After the large glass plates in the 19th century, the roll film industry started a miniaturization in cameras, e.g., the Rollei 6 x 6 cm reflex, the 1:1 ratio. In the first Leica cameras, the even smaller 35 mm movie film was used, with a 24 x 36 mm size of negatives, the 2:3 ratio.

TABLE 1. Aspect Ratios in Movies, Stills Photography, and TV

Rectangle	Height by Width Ratio a:b	a:b	b:a	Diagonal Factor K= $\sqrt{a^2/b^2 + 1}$	Usage
	1 : 1	1.000	1.000	1.414	square, 6 x 6 film camera, 1920's
	3 : 4	0.750	1.333	1.250	early TV, early PC monitor, early digital stills camera
	1 : 1.37	0.730	1.370	1.238	“Academy” movies (before 1950's)
	2 : 3	0.667	1.500	1.202	35 mm film stills camera (1920's)
	9 : 16	0.563	1.778	1.147	APS camera 1996, and the 2000's widescreen TV, PC monitor, digital stills & movie camera
	1 : 2	0.500	2.000	1.118	Superscope movies (1950's)
	1 : 2.35	0.426	2.350	1.087	Cinemascope movies (1970's)

For large-screen life-size cinema, where the spectator almost becomes part of the action, there is more reason to copy the wide horizontal viewing angle of the human eye. This trend is seen in the evolving aspect ratios of the black & white movies, Superscope, and Cinemascope successively, and is still continuing.



FIGURE 2. Horizontal Bars on TV for a Wide Screen Movie

In the world of TV, we also see a trend from the original 2:3 ratio towards greater width.

The 9:16 aspect ratio of today’s wide-screen TV was a compromise of many competing wide-screen formats before 2000, but not as wide as real cinema, so a movie image has to be cropped or zoomed onto a 9:16 screen, see the Figure 2.

For PC screens, the same trend has occurred. Early monitors were 2:3, some 17 inch screens even approaching 4:5.

Windows by Microsoft was not very suitable for wider screens, because of their horizontal menu bars or “ribbons” (although those can be rotated into verticals). Still the desire to view wide-screen movies on the PC caused the monitor screens also to expand to the 9:16 format, around 2010.

Printing Paper Aspect Ratios

A sheet of paper is another viewing frame, used in a standing position in order to read successive lines of text by scanning from top to bottom. Here the text lines should not be too wide, to minimize horizontal eye movements. Table 2 shows some main examples of current sheet paper aspect ratios (ancient formats such as *quarto*, *octavo* are a different story).

The interesting aspect ratio of $\sqrt{2}$ in the international ISO standard range of formats A0, A1, A2, A3, A4, etc. stems from the requirement that each format size can be obtained by halving the preceding format, expressed in the formula: $a:b = b:2a$. For example the format A4 (210 by 297 mm) is obtained by halving A3 (297 by 420 mm).

Note that halving of 2:3 aspect ratio paper sheets (such as classic photographic paper) results in 3:4, and vice versa!

Other Aspect Ratios

Of course there have been, and there still are, many more aspect ratios than the main examples given above, especially in the history of cinema and in paper sizes.

A very different field of aspect ratios is found in the figurative arts: paintings, drawings, engravings, etc. Here the artist, or his patron, determines the size of the work of art. Any aspect ratio is possible, there is no standard.

However, there was one exception: the Golden Section, also known as Golden Ratio or Divine Proportion, was already known in Greek antiquity, and at that time considered the most beautiful proportion in architecture and other works of art. Leonardo Da Vinci considered aspects of the human body conform to the Golden Ratio, see Figure 3.

The Golden Ratio for the lying rectangle with height “a” and width “b” was defined as $a:b = b:(a+b)$, resulting in the ratio $2/(1+\sqrt{5}) = 0.618$. This positions the Golden Ratio between the ratios 2:3 and 9:16 in Table 1.

After the Renaissance, the Golden Ratio became of less importance, although some artists continued to use the Golden Ratio, mostly for sections within their paintings.

TABLE 2. Aspect Ratios in Sheet Paper

Height by Width Ratio a:b	a:b	b:a	Diagonal Factor K = $\sqrt{(a^2/b^2 + 1)}$	Usage
8.5:11	0.773	1.294	1.264	Letter (US standard paper sheet, lying)
1: $\sqrt{2}$	0.707	1.414	1.225	A0, A1, A2, A3, A4 etc. (ISO standard paper size, lying)
8.5:14	0.607	1.647	1.170	Legal (US standard paper sheet, lying)

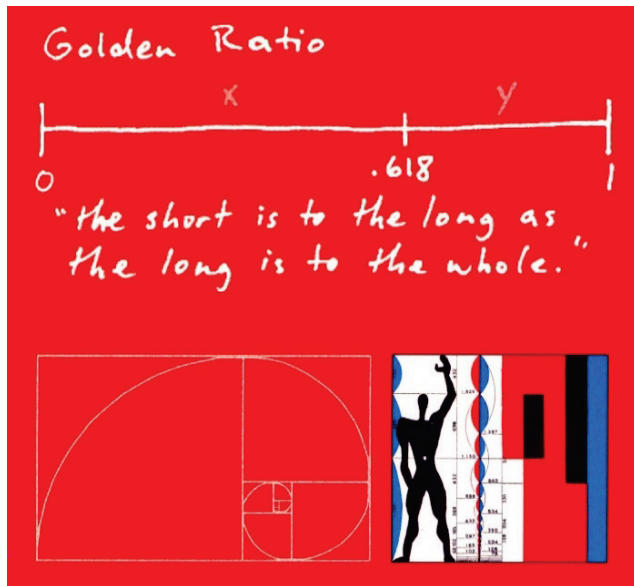


FIGURE 3. Golden Ratio by Da Vinci

Panorama Views

An example of extreme wide formats is the “panoramic” view, in 1996 already part of the APS film cartridge standard, with a 1:3 height to width ratio.

Special cinema installations exist for half-circle projection, e.g., IMAX Dome, OMNIVISION.

The ultimate form of a panoramic view is the 360° field, where spectators have to turn their heads for a complete view, like in the real world. Many museums have their own movies

in full-circle or in rooms with 100% full-wall projection, for example the *Voyage at Sea* experience during a raging storm, in the restored and updated Amsterdam maritime museum, *Het Scheepvaartmuseum*.

Artists have created full-circle panoramas, for example the “Metamorphosis II” (19.2 by 389.5 cm, ratio 1:20.3, see Figure 4) by Dutch graphical artist M. C. Escher, that can be turned into an endless band with a diameter of 1.24 m, to make a mini-panorama around your head.

A larger example is the life-size “Panorama Mesdag” (14 by 120 m, ratio 1:8.6, see Figure 5) of *Scheveningen*, 1880, by Dutch painter H. W. Mesdag, with a gazebo for spectators on an artificial dune, in the center of the 38 meter diameter.

Ratio Calculations by Slide Rule

Adjacent logarithmic scales on body and slide of a slide rule are the perfect means for setting an a:b ratio from Table 1 or Table 2, and reading any a:b combination of height “a” and width “b” without ever touching the slide rule: the ideal “hands-free” operation.

The A and B scales are the easiest to use for hands-free reading of a:b, because the double decade allows the full range of a and b to be available at any setting of a:b. This is not the case for the C and D scales, where portions of the C scale dangles in the air, depending on which side the slide has been pulled out. However, if there is a CF/DF scale pair available above the C/D scales, the hands-free operation is just as easy as on the A/B pair, with the added advantage of more precise or better readable scale markings.



FIGURE 4. Metamorphosis II by M.C. Escher



FIGURE 5. Panorama Mesdag

Calculation of the diagonal “d”, given “a” and “b”, is not so easy on the slide rule. There are a number of methods for the Pythagorean formula $d = \sqrt{a^2 + b^2}$. Using the A/B and C/D scale pairs for squares and roots still requires - under the square root sign - the manual addition of $a^2 + b^2$, or of $a^2/b^2 + 1$.

Another method is via trigonometric functions, by the formula $d = a/\sin\{\arctan(a/b)\}$, which also requires extra work: the laborious reading and manual copying of the diagonal’s angle from tangent to sine scale.

Some slide rule makers have designed separate Pythagorean scales, for example the P and Q scales by Sun-Hemmi in Japan, the Pt scale by Thornton in the UK, or the short M-scale by Skala in Poland.

However, for the purpose of predefined ratios we need not calculate the diagonal, because we can use a pre-calculated factor $K_{a:b} = \sqrt{a^2/b^2 + 1}$ in the formula: diagonal $d = b \cdot K_{a:b}$. These constant factors $K_{a:b}$ are given in Table 1 and 2 for all ratios listed.

If, for example, we are only interested in the ratios 3:4 and 2:3 and 9:16, we could use three gauge points $K_{a:b}$ on the appropriate positions at the A and B scales (or at the C, D, CF, and DF scales).

Then we can swiftly calculate, after setting the slide to the proportion a:b, the following:

1. diagonal d from height a $d = a \cdot K_{a:b}/(a:b)$
2. diagonal d from width b $d = b \cdot K_{a:b}$
3. height a from diagonal d $a = d \cdot (a:b)/K_{a:b}$
4. width b from diagonal d $b = d/K_{a:b}$

The Easy Method by Slide Rule

The gauge mark method described above still needs multiplication or division, with possible confusion between the two. The easier method is to mark the diagonal constants $K_{a:b}$ as gauge marks on the cursor. This is possible because all diagonal constants are within the range 1.000 to 1.414. On most slide rules the maximum factor on the cursor between hairline and right side of the glass reaches a factor of about 1.4 on the A and B scales. The cursor range on C and D scales is much less, so we will not consider C and D.

In Figures 6 and 7 below, an Aristo Darmstadt 967U is shown, with three K gauge marks in green added to the cursor by the present author. The gauge marks are chosen for the photography-oriented ratios 3:4 ($K_{3:4} = 1.25$), 2:3 ($K_{2:3} = 1.202$), and 9:16 ($K_{9:16} = 1.147$).

Notes

1. Note that this method assumes the smaller ratio value (height a) to be on the A scale, and the larger one (width b) on the B scale. If these two were to be reversed, the arrow mark gives the diagonal result on the A instead of on the B scale (behind instead of under the arrow).

The 967U model was chosen because the cursor is relatively wide, and because there is enough empty space on the cursor at the A and B scales height to add gauge marks and explanatory texts.

If the slide is set to a different ratio, say 9:16, then the leftmost green arrow $K_{9:16}$ on the cursor can be used likewise for diagonal calculations of any combination of a and b in the ratio 9:16.

Conclusion

Frames for viewing pictures can have a wide variety of aspect ratios, from very high and narrow (consider peeping through a fortification's arrow slit) to very wide and low (consider peeping through a horizontal mail slot).

Current technology for watching TV or computer screen favours mostly the aspects ratios of 3:4, 2:3, or 9:16.

Calculations between height, width, and diagonal sizes of such viewing frames can be done very easily with a regular slide rule, on which just three additional gauge marks have been added to the cursor.

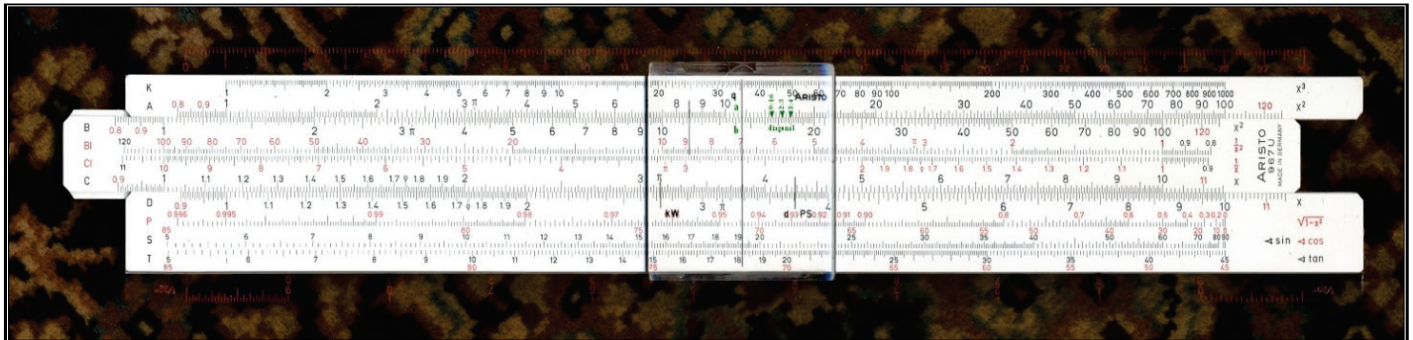


FIGURE 6. Slide Rule with Aspect Gauge Marks on Cursor

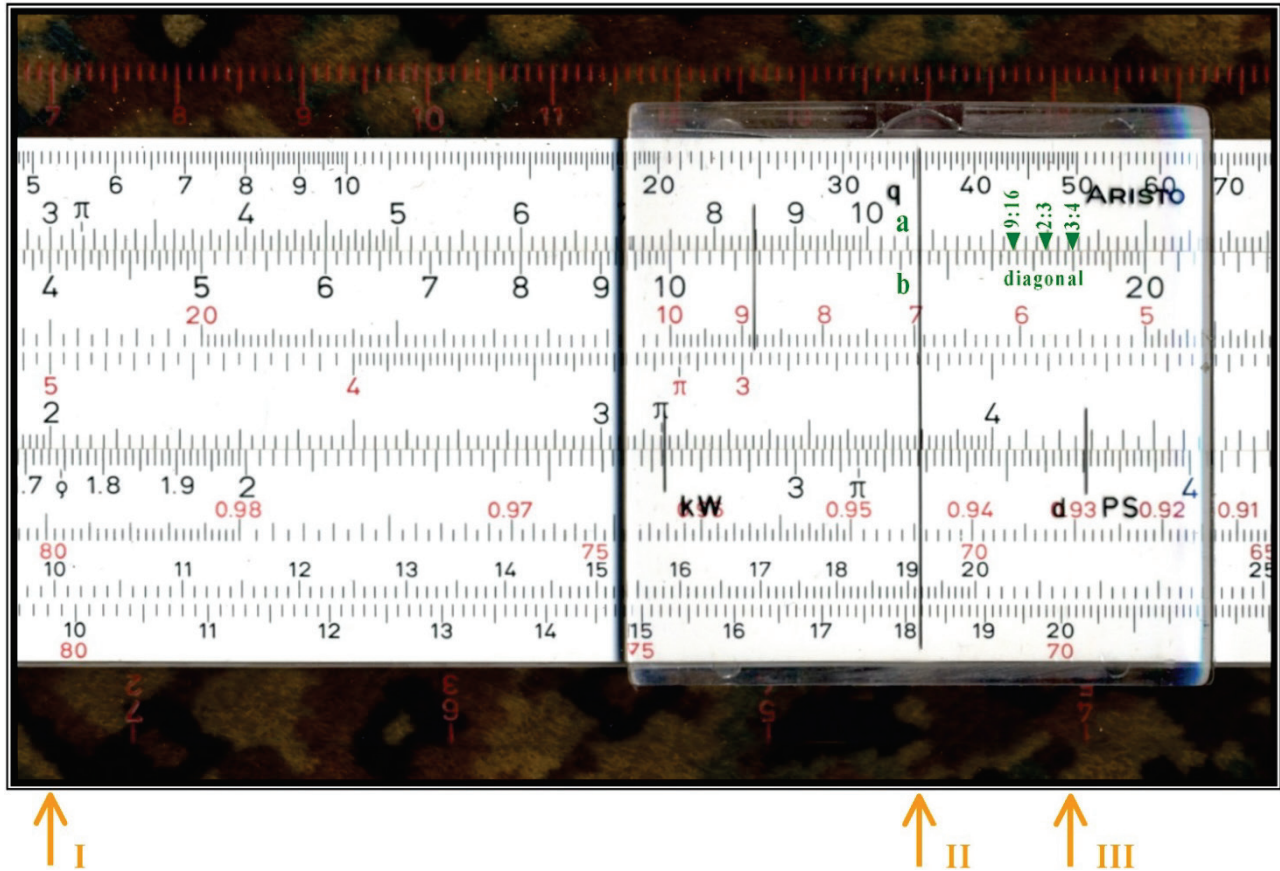


FIGURE 7. Aspect Gauge Marks on Cursor – Details

- I. The slide is set to the ratio $a:b = 3:4$ on scale A and scale B respectively, see the orange arrow I. The slide should be kept in this position during all calculations for the ratio 3:4. Any a and b combination for the ratio 3:4 can then be read hands-free on A and B scales, for example 6 by 8, 9 by 12, etc.
- II. Keeping the slide at the 3:4 ratio, the cursor is now moved to $a:b = 10.8:14.4$, on scale A and scale B respectively, see the orange arrow II. The K-factor for ratio 3:4 is the rightmost green arrow on the cursor, corresponding with $K_{3:4} = 1.25$ on the A scale.
- III. The diagonal $d = b \cdot K_{3:4}$ can now be read below the $K_{3:4}$ gauge mark on the B scale, resulting in $d = 18.0$, see the orange arrow III. By moving the cursor, while keeping the slide at the 3:4 ratio, any combination of height a, width b, and diagonal d can be shown, using the rightmost green mark under the cursor.¹