FIGURE 6. Front View of the Vrégilles Disc

The Small Multiplication Table through the Centuries in Europe

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Nowadays the small multiplication table up to 9*9 or 10*10 only serves as an educational tool. In former times with less schooling and therefore less practical knowledge, the table was also used as an aid for multiplication.

This article will show some tables in their various forms over centuries and at the same time will identify other aspects of their appearance such as language, numerals, and the technology of printing.

Due to the different historical developments in the New World and in the Old World, the scope of the shown examples is restricted to Europe.
Although the complete small multiplication table includes 100 partial products from 0*0 to 9*9, the products with 0 were always skipped and those with 1 often because of their simple results, which can be memorized easily.

In Medieval tables with Roman numerals, we find 9 as well as 10 for the upper limit of factors. Later writers adopted these two limits for the small multiplication table, although the factor 10 in Arabic numerals is actually not a single digit but a compound number.

In a small multiplication table, both product factors cover the range from 1 or 2, to 9 or 10, independent of one another. Therefore, the arrangement in the form of a square is obvious. One of the two factors is written on the upper side and the other one on the left side of a square. Where row and column meet the product can be found. Figure 1 shows an early table from end of 10th or begin 11th century, probably the Upper Rhine region. The table is hand written using Roman numerals and is one of those rare items with artistic design. Not only did the writer weave the colored border lines, he also marked the diagonals with small flowers.

Up to the early 13th century religious books, histories, legal or teaching texts including geometry and arithmetic, were written in monastic *scriptoria* (writing rooms or offices) and in most cases in Latin, the language of scientists.

Due to the commutative law a*b = b*a, the multiplication table can be shortened, covering only the products 2*2... 2*9, 3*3... 3*9, 4*4... 4*9, up to 9*9 or 10.

In Figure 2 we see the first part of another form of a multiplication array, the list. This is a shortened list. The first line reads *1 semel unus unus est et unus digitus* (“1 one times one is one and one unit“) and continues with 1*2 = 2, 1*3 = 3, and so on up to 9*9. The last line, not displayed here, says *9 novies novem LXXXI octo articuli et II digiti* (nine times nine 81 eight tens and 2 (!) units). Obviously writing digits or multiplication was not easy during those times.

The table in Figure 2 was written about 1040. To ease searching the appropriate line, the table is marked on the left side with an *apex* (plural *apices*). These use an early and artificial form of Arabic numerals, which were initially introduced to Europe through Spain about this time.

The arrangement in Figure 3, in a manuscript from the 12th century, is also a shortened list with a two-dimensional arrangement. From top down, the small squares read:

> *Semel I I* (one times 1 1),
> *Bis II IIII* (two times 2 4),
> *Bis III VI* (two times 3 6),
> *Ter III VIIII* (three times 3 9),

and so on. Since the arrangement has no highlighted entries like others, the user is not directed to the result, he has to read or count the squares to obtain the product.

The author of one of the oldest printed books on mercantile arithmetic from about 1471/1482 gives calculating examples for merchants in Germany. To address these people he wrote in the German language instead of Latin. The multiplication table he added is arranged in a shortened list and covers two pages. Figure 4 shows the left half. Now we find our modern Indo-Arabic numerals although the digits 4, 5, and 7 are in an early shape. The way this book has been printed is worth being mentioned. It is called a block book, because instead of using movable letters they cut one or two pages mirror-inverted as a whole into a wooden block and then print it.

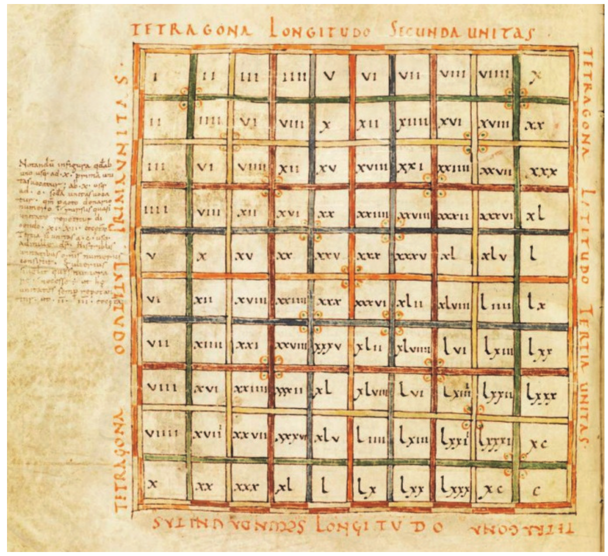

FIGURE 1. Hand Written Square Table with Roman Numerals, about 1000

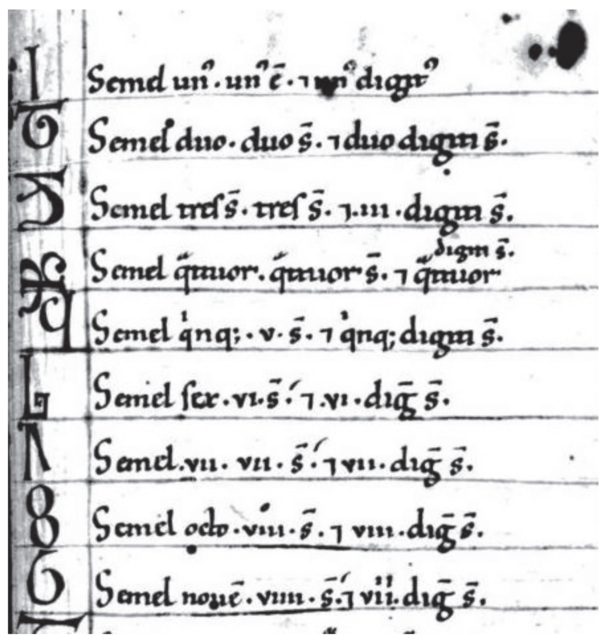

FIGURE 2: First Lines of a Shortened List with Apices from about 1040

Sometimes lists are arranged in specific forms as shown in Figure 5 without and in Figure 6 with written arithmetic operation support.

The multiplication table up to 9*9 or 10*10 is part of almost all books on arithmetic. The authors insist on learning the table and use rhymed short sentences like (in free translation) ″study the multiplication table and you will become familiar with all kind of calculation″ [1] or ″who intends to become a good calculator has to learn the multiplication table″ [2] and similar. Furthermore, they sometimes give a detailed explanation on how to use the table for those readers who have not memorized but use the table as a tool for multiplication.

If one shortens the quadratic arrangement, one half is missing and the shape changes to a triangular multiplication array. Those arrangements appear in various forms. In Figure 7, the author only uses one half of a square, the diagonal line with square numbers is explicitly marked with *Quadrati numeri*. In Figure 8, the left side and diagonal line serve as two entries, in Figure 9 only one highlighted entry to the table is available.

Except the used digits – Roman, *apices*, Hindu-Arabic – there is no development in the shape of arrangement during centuries. All forms of the small multiplication table – square, triangular, and list, complete or shortened – are in use at the same time.

From the late 18th century, the multiplication table was used as an educational tool, as well a toy for children. As an example, the square table in Figure 10 with vertically movable entry "window" is part of a toy named *Tables Intuitives Reumont* from France, early 20th century.

Finally I introduce the unknown owner of a trigonometric table, published in 1639, who wrote his own shortened multiplication list on the endsheets of his book (see Figure 11).

I do not know what this table was used for, maybe for learning, maybe as an aid for multiplication. He captioned his table with *Tabula Pythagora*, because at that time people erroneously regarded the Greek mathematician Pythagoras to be the inventor of tables of that kind. At the end of his table he did not forget to add the word *FINIS* (the end).

References

1. Widmann, Johann, *Behend und hüpsch Rechnung uff allen Kauffmanschafften,* Pfortzheim 1508.
2. Lammerding, Johann, *Die selbst-lehrende Rechen-Schule,* Münster 1718.

Figures

Sources and abbreviations for the owners:

StBBa: Staatsbibliothek Bamberg, Germany.
BStBMu: Bayerische Staatsbibliothek, Munich, Germany.
Figure 1: Anicius Manlius Severinus / Isidorus <Hispalensis> / Hieronymus, Sophronius Eusebius: *Boethius, De institutione arithmetica [et al]* - StBBa Msc.Class.6, [S.l.], fol. 38v.
Figure 2: Johannes Scotus Eriugena – Dionysius Areopagita. BstBMu Clm 14137, fol. 113r (Img 00228).
Figure 3: *Bedae libri de arte metrica fragmentum*... BStBMu Clm 14689, fol 50r (Img. 00102).
Figure 4: Ulrich(?) Wagner, *Regula von dre ist drey dinck die du setzt,* (about 1471/1482). StBBa Inc.typ.Ic.I.44, fol. 1v and 14r.
Figure 5: Adriaan Metius, *Arithmetica et Geometria,* 1640.
Figure 6: William Leybourn, *Arithmetick, Vulgar, Decimal, Instrumental, Algebraical: In four parts,* London 1700.
Figure 7: Rainer Gemma Frisius, *Arithmeticae Practicae Methodus Facilis,*1561.
Figure 8: Petrus Apian, *Eyn underweysung aller Kauffmanß-Rechnung,* Ingolstadt 1527.
Figure 9: Tobias Beutel, *Neu aufgelegte Arithmetica oder sehr nützliche Rechen-Kunst...* Leipzig 1678.
Figures 10, 11: from the author's collection.
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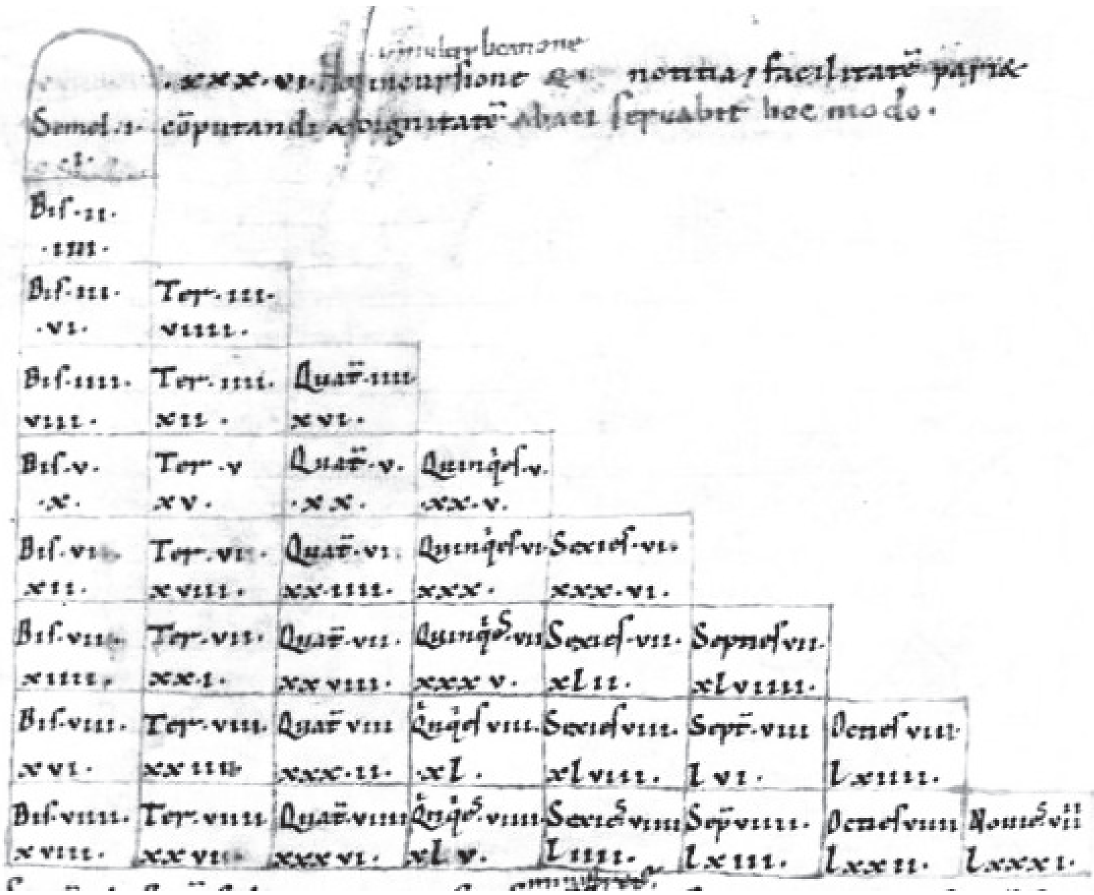

FIGURE 3. Hand Written List Array, 12th Century

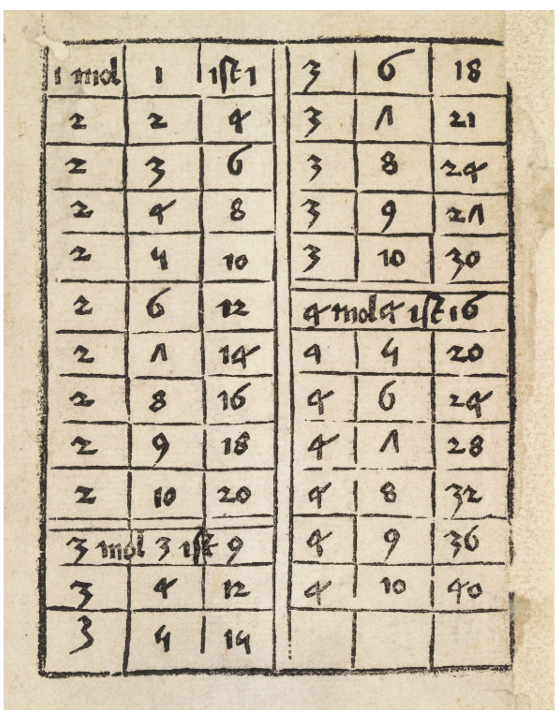

FIGURE 4. An Early Printed List (left half) in a Book on Mercantile Arithmetic from 1471/82

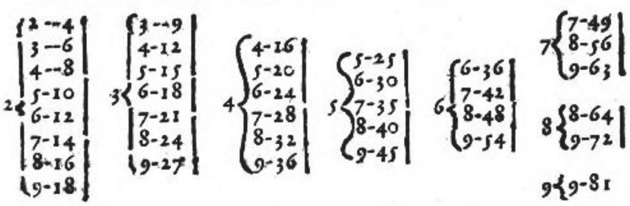

FIGURE 5. Multiplication List in a Book from 1640

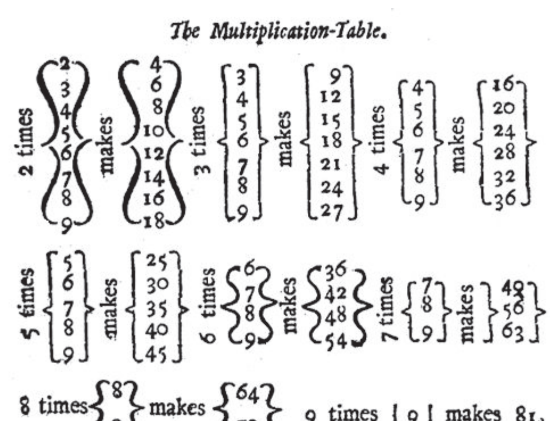

FIGURE 6. Multiplication List in a Book from 1700

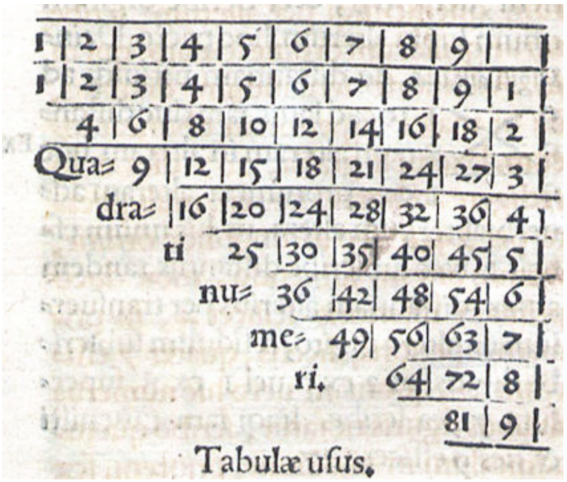

FIGURE 7. Triangular Table in a Book from 1561

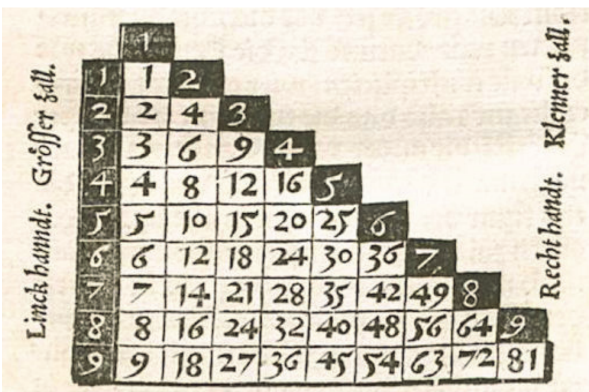

FIGURE 8. Triangular Table in a Book from 1527

FIGURE 9. Triangular Table in a Book from 1678

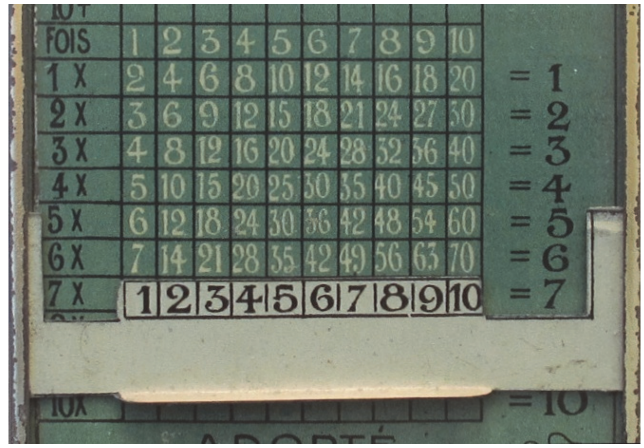

FIGURE 10. Square Table with Movable Entry, System *Reumont*

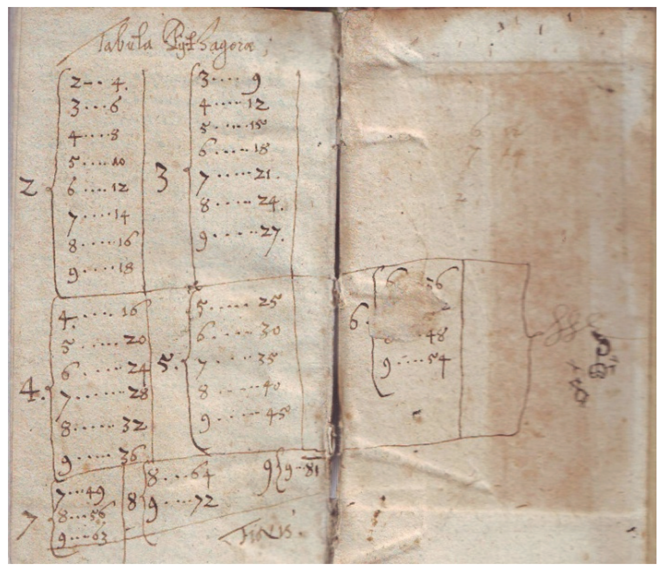

FIGURE 11. Hand Written Shortened List, about Middle of the 17th Century