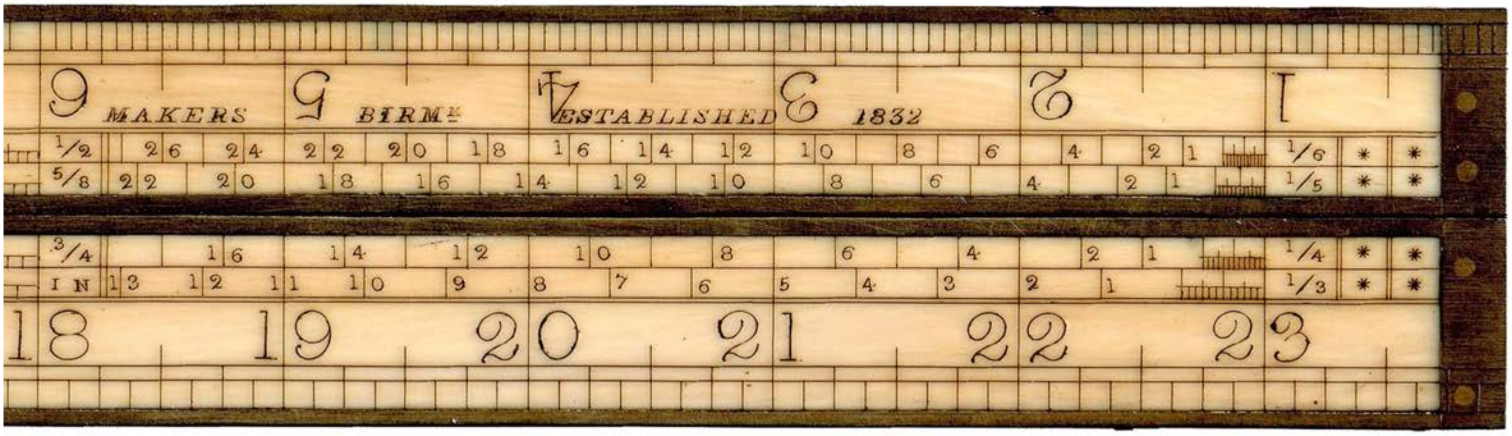

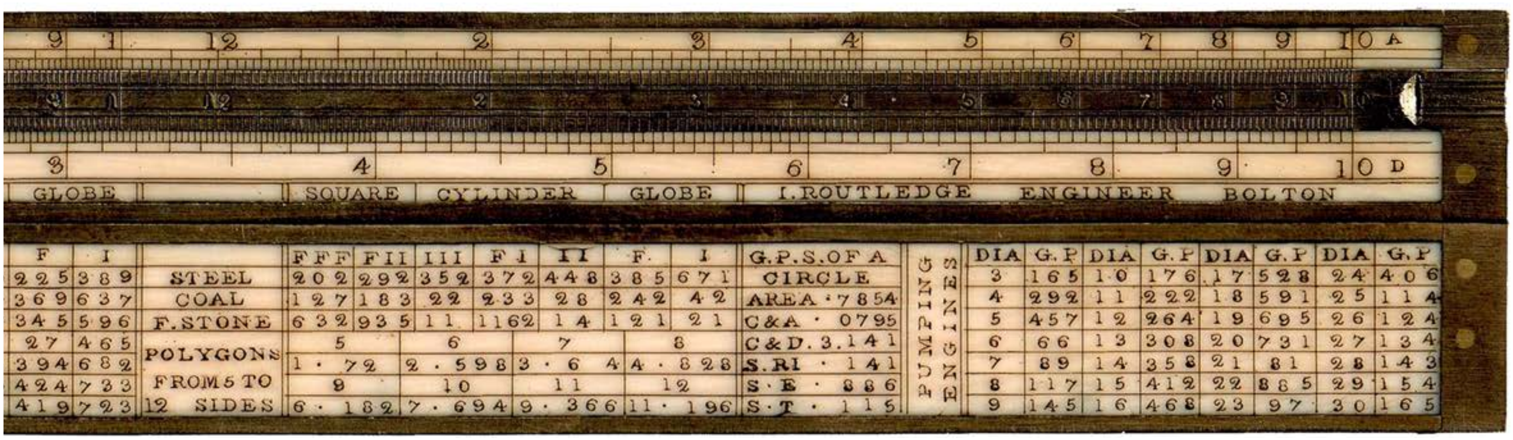

Wang LOCI Calculators[1] – Electronic Calculating by Logarithms

Peter Holland & Otto van Poelje

Introduction

Dr. An Wang (1920 - 1990), of Chinese origin, became a famous American, an inventive engineer in digital electronics (holding some 40 patents) and a successful entrepreneur in the computer industry, see [2]. In 1951 he founded his own company Wang Laboratories in Boston, with a patent on magnetic core memory control as intellectual capital. After a long struggle with IBM about that patent, Wang Laboratories started late in the 1950s earning solid money with their innovative typesetting machine named Linasec.

In the early 1960s, Wang turned his attention to calculators as a new business opportunity. At that time the calculator world was in full transition from mechanical to electronic implementation. Electronic tabletop calculators had already been brought to market, although expensive and often built with less reliable switching devices such as telephone relays, radio tubes or thyratrons. But solid-state calculators – with discrete transistors – started to appear in the mid 1960s, e.g., the Friden EC-130 (1963) or the Mathatron (1964).

Principles of the Wang Calculator

Wang's goals for a new calculator were set to make it competitive with mainframe computers for calculating more complicated algebraic formulae than mere multiplications and divisions. It had to be cheaper and easier to use than mainframes, but of comparable speed in that type of calculations.

The mainframes of that time and the newer mini-computers then starting to come out used full word-length for representing numbers in the CPU (Central Processing Unit, then consisting of many racks of printed circuit boards in stead of today's single CPU chip).

The binary numeral system represents numeric values by using only two symbols, 0 and 1. A CPU "word" with a length of for example 32 bits could represent 2^{32} values (4,294,967,296), giving a precision of 9 decimal digits. For higher precision numbers double-length words were needed, with additional micro-programs. Binary arithmetic reduces every operation to simple additions or shifting of bits within a word, e.g., exponentiation is first reduced to multiplication and then to addition which can easily be performed in binary. However, Wang chose for the decimal representation to use 4 bits for each decimal digit (so-called BCD or Binary Coded Decimal), not because it was since long customary for calculators, but primarily because he needed this to realize his other design decision: calculating by logarithms.

This logarithmic foundation of the Wang calculator makes it a most interesting stepping stone from logarithmic table and slide rule to the digital calculator.

Logarithms by Successive Approximation

Wang's electronic calculator has been the only one designed to perform arithmetic operations by logarithms; hence, the name LOCI (LOgarithmic Calculating Instrument). The LOCI performed calculations by using natural logarithms. The idea to use logarithms in an electronic calculator was not easy to realize in the 1960s. A complete table of natural logarithms should be stored in the memory of the device, which was impossible due to the high prices of memory in the calculator.

So, Dr. An Wang and Frank Trantanella - who later became Vice President of Wang Laboratories - developed a transistor logic, which could calculate the natural logarithm of any numeric value by using only 6 (six!) natural logarithm values stored in the ROM of the calculator. Details can be found in US Patent 3,402,285 from September 1964, see [7]. The most important sentence of this patent is:

The calculator also includes a store of logarithmic values of the following constants: 10, 2, 0.9, 1.01, 0.999 and 1.0001

The principle of "successive approximation" is used for calculating logarithms (in the patent Wang calls it the "factor-combining" method). The sequence of approximation steps is not determined mathematically as in the "Regula Falsi" or the "Newton-Raphson" method, but purely by the direct implementation in digital registers, by multiplication/division via shifting and adding digits in registers and by control logic to determine the flow of bits and selection of each of the constant factors.

The choice of the numerical value of the factors will be addressed in the next section of this article.

The factors 10 (or 0.1), 2, 0.9, 1.01, 0.999 and 1.0001 form a sequence of decreasing step sizes in the approximation towards the value 1 of which the logarithm is zero.

The function to be solved can be written as:

$$\ln(N) = x$$

where N is the value of a number used in calculations, and x the natural logarithm of N. Via approximation steps of multiplying N successively with delta's (Δ) of decreasing size, the value N is moved towards 1, resulting in:

$$\ln(N \times \Pi\Delta) \rightarrow \ln(1)$$

or:

$$\ln(N) + \Sigma \ln(\Delta) \rightarrow 0$$

Therefore the required logarithm is approximately:

$$x = -\Sigma(\Delta)$$

The approximation is converging because the logarithm is a strictly increasing function. This is a very clever idea because, after the approximation has reached the required precision of the value 1, the sum of the logarithms of all delta multipliers determines the logarithm *x*.

The following numbers are stored in the control unit of the LOCI calculator:

TABLE 1.
Multiplication Constants

Δ	$\ln(\Delta)$
10	2.30259
0.1	-2.30259
2	0.69315
0.9	-0.10536
1.01	0.00995
0.999	-0.00100
1.0001	0.00009

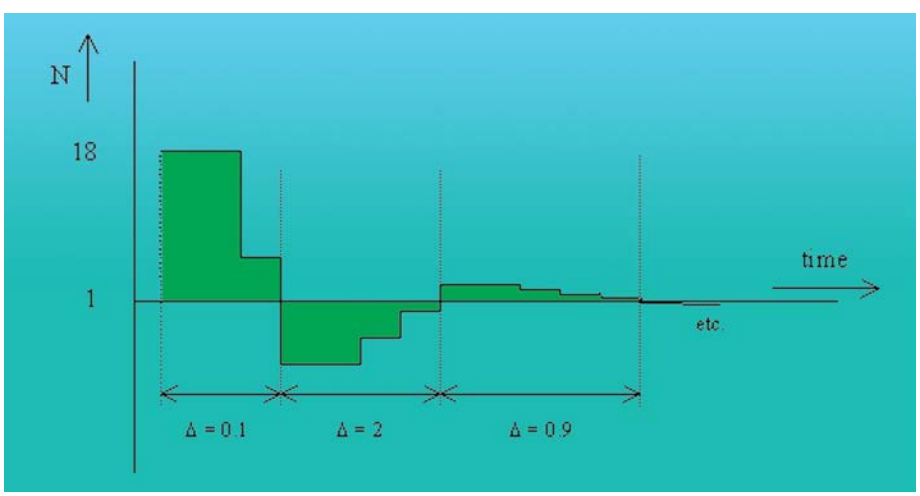

FIGURE 1.
Three Approximation Cycles

How did the calculation of a natural logarithm work?

It is demonstrated here with the example of the decimal value of 18. The value 18 must be multiplied successively with the constants stored in the memory of the LOCI (10 or 0.1, 2, 0.9, 1.01, 0.999, 1.0001) to approximate the decimal value 1. As 18 is greater than 1, it first has to be multiplied repeatedly by 0.1 until it is for the first time smaller than 1, then it has to be multiplied repeatedly with 2 until it is for the first time greater than 1, and so forth.

Note that during the approximation the value of N is alternating per step size between smaller than 1 and greater than 1, see table 2 for the values during the actual approximation and the sketch in figure 1 to visualize the process (the vertical axis in the sketch is non-proportional, but not strictly logarithmic either).

The correct answer is ln (18) = 2.8903717…, so in this example, the approximation of the logarithm has an accuracy of 5 decimals. In the first commercial model, the LOCI-2, the precision of user results was 10 digits (see the 10 Nixie tubes in fig. 3). To reach that precision, the internal register length must have been larger than 10 digits and the number of logarithmic factor values must have been greater than the 6-fold in the patent description.

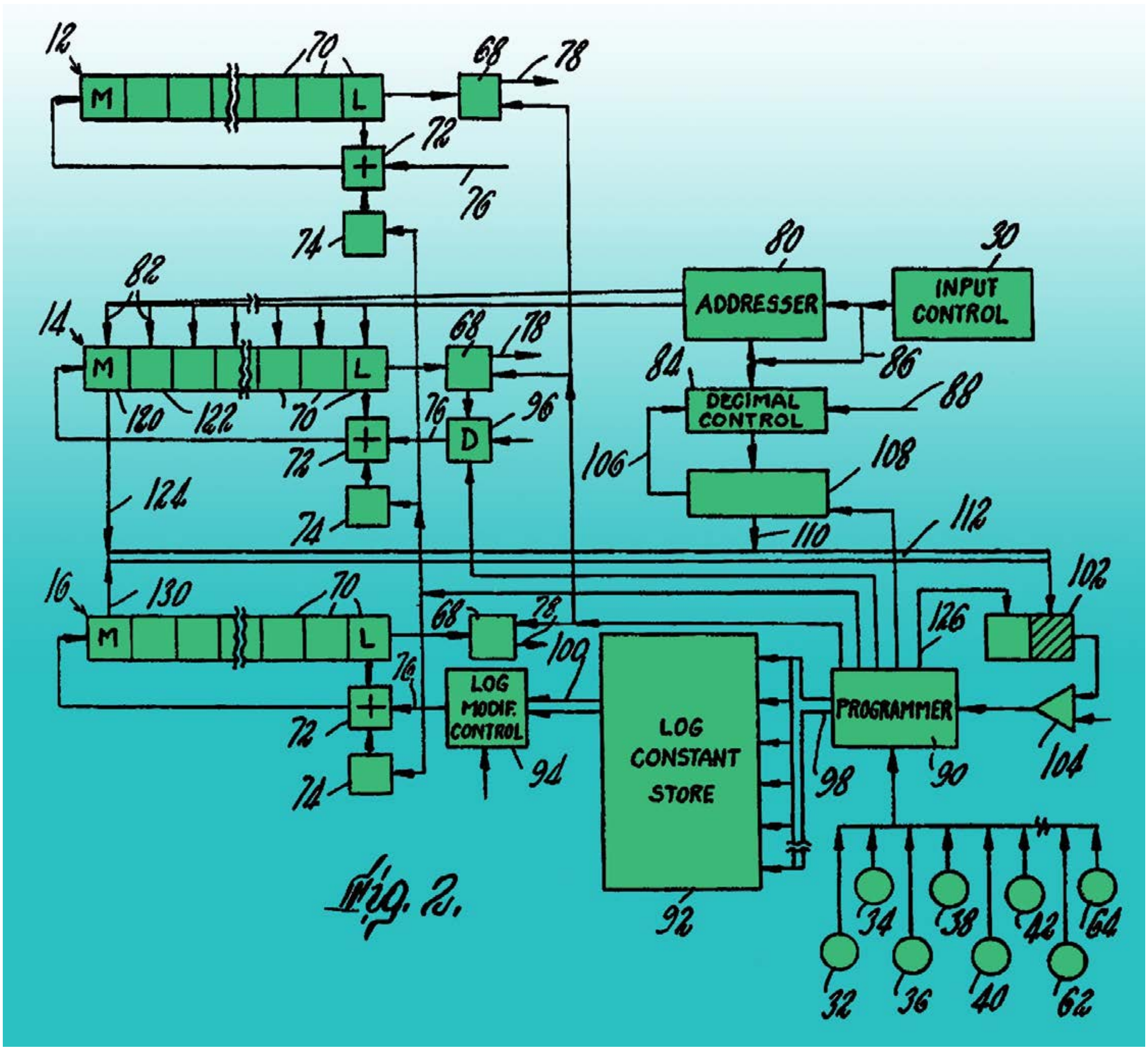

FIGURE 2.
Logic Diagram in Patent

TABLE 2.
Approximation steps for ln (18)

N during approximation	multiplier Δ	nr of steps x Δ	next N in the approximation to 1		ln (Δ)
18	10	0	18	(>1)	0
18	0.1	2	$18\times(.1)^2 = 0.18$	(<1)	2×-2.30259
0.18	2	3	$0.18\times 2^3 = 1.44$	(>1)	$3\times +0.69315$
1.44	0.9	4	$1.44\times (0.9)^4. = 0.944784$	(<1)	4×-0.10536
0.944784	1.01	6	$0.944784 \times (1.01)^6 =$ 1.0029072	(>1)	$6\times +0.00995$
1.0029072	0.999	3	$1.0029072\times (0.999)^3 =$ 0.9999015	(<1)	3×-0.00100
0.9999015	1.0001	1	$0.9999015\times 1.0001 =$ 1.0000015	(>1)	$1\times +0.00009$
1.0000015			$x = \Sigma \ln(\Delta) = 2.89038$		

Hardware Implementation

Wang's patent describes the method for calculating the logarithm and the hardware design for that method. Figure 2 shows the logic diagram.

The data registers are:
12: accumulator
14: input register
16: logarithmic register

The crux of the approximation method lies in the fact that some of the multiplications with the factors in table 1 are split into elementary *add* and *shift* operations in the BCD registers:[2]

TABLE 3.
Dissection of Multiplication Factors

Δ	*add & shift*
10	10
0.1	0.1
2	1 + 1
0.9	1 – 0.1
1.01	1 + 0.01
0.999	1 – 0.001
1.0001	1 + 0.0001

Every approximation factor has an add/subtract component, and a multiplier that is a power of 10; adding is a fast parallel operation (72) and multiplication of a BCD numbers with a power of ten is executed very fast by shifting the decimal point in the register.

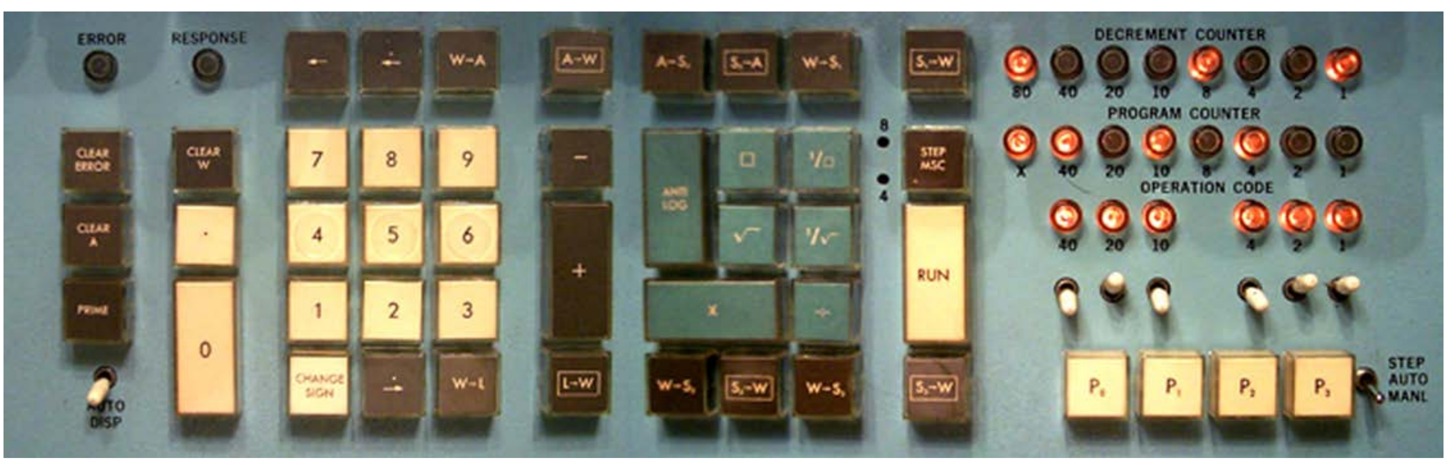

FIGURE 4.
LOCI-2 Keyboard

While the factors are applied to the number in input register (14) – by adding and shifting – to approximate the value 1, the log register (16) is adding simultaneously the corresponding values from the log constant store (92), to reach eventually the required logarithm. The speed of this “hard-wired” operation is high because there is no need for machine-instructions and micro-code as used normally in general computers. With 6 factors the maximum number of clock-cycles (approximation steps) is never more than 6 x 9 = 54 (with an average of 27).

The accumulator register (12) was used for adding and subtracting numbers in the logarithmic domain, to achieve multiplication and division.

To calculate the anti-logarithm (after having added or subtracted logarithmic values) the same circuits are used with small changes in control.

FIGURE 3.
Wang’s LOCI-2

Commercial Success

The first successful product of the LOCI calculator series was LOCI-2 with a higher precision than its predecessor LOCI-1, see [3] and figure 3.

Compared with later calculators, the user-friendliness was debatable, looking at the keyboard in figure 4. In addition to the usual arithmetic keys (including inverse, square, and square root, and using Reverse Polish Notation) many less-intuitive keys were present for manipulating registers and programming the calculation of complete formulae.

Such formula-defining programs could be entered on punched cards for which a special reader unit had to be connected to the LOCI-2. This important feature of programmability was emphasized in the marketing, see figure 5 from [8].

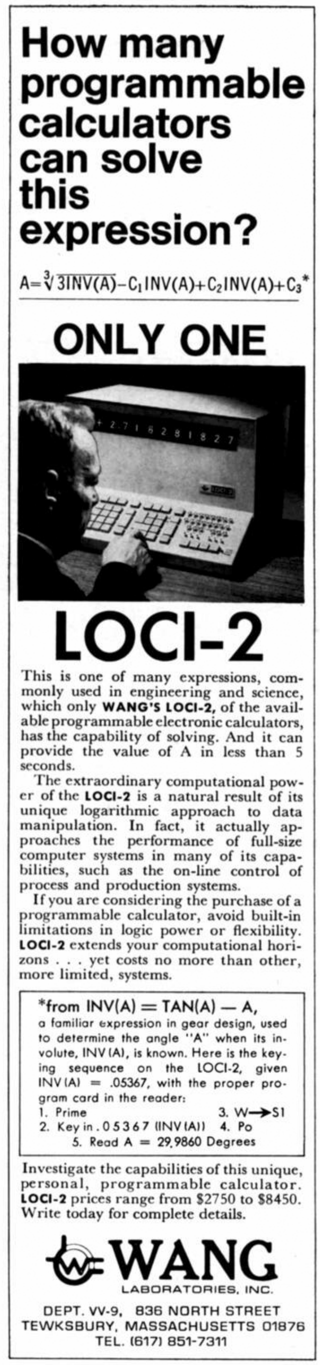

FIGURE 5.
LOCI-2 Ad in Scientific American

As successor to the LOCI-2, the 300-series (1966) offered many important improvements, see [4]. A rounding-off circuit was added to remove the annoying phenomenon of showing calculation results like 6 x 8 = 47.99999..., caused by the approximation of the logarithms that were used for the multiplication. But more importantly, the calculator was split up in a (suitcase-sized) processor unit and separate keyboard/display units, see figure 6, with not much more than the regular arithmetic keys needed by a user on a calculator.

FIGURE 6.
Keyboard/Display Unit of 300-Series

The 300-series was brought to market in many different versions, of which for example the 360-KT had additional trigonometric functions under dedicated keys (implemented by an internal program hard-wired in a diode matrix), and the 370/380 had added conditional branching to programs on punched card (370) or magnetic tape cartridge (380). Up to 4 keyboard units could be connected in time-sharing mode to one processor unit of the 300-series "SE" (Simultaneous Electronics). Also a series of external devices were introduced e.g., printers, tape units, core memory extensions. The much improved 300-series was a big success in the market, not in the least because the prices tumbled down from $6,700 for the first LOCI-2 in 1965, to $1,700 for a basic 300-system in 1967. The 300-series was succeeded in 1970 by Wang's new 700-series as direct response to the release of the innovative HP-9100A by competitor Hewlett Packard. The Wang 700 still used the successive approximation method, but only to calculate pure logarithms and anti-logarithms; its multiply/divide operations now used conventional repeated additions/subtractions.

Conclusion

The LOCI series of desktop calculators by Wang Laboratories (LOCI-2 and the 300-series) were a success in the calculator market, and kept the company buoyant until even greater successes were achieved in other calculators, word processors, minicomputers, and ICT in general.

The design principle of calculating by logarithms was unique for electronic calculators, and was only made possible by a very clever hard-wired implementation.

Acknowledgments

Rick Bensene has published the most extensive information on Wang (and other) calculators at the Old Calculator Museum website, see [3], [4], [5]. He has kindly provided and authorized the photographs for this article. We also thank him for his advice and for his review of this article.

Notes

1. Adapted and extended from Peter Holland's original presentation at RST Miltenberg, 2005 [1]. JOS Plus indicates that additional material for this article is available on our website www.oughtred.org. For this article a copy of the US patent [7] is included.
2. The patent describes in column 4 (page 2) that:

the last 4 multipliers are of the form $1 \pm 1/R^A$ where R is 10 and A is successively the integers 1, 2, 3 and 4.

Actually the factor 2 also fits in this formula with A = 0. The formula however excludes the first factor (dividing by 10 or multiplying by 10).

We believe it very likely that the digital designer Wang first constructed his approximation with decreasing factors in efficient logic circuit operations, and only post-hoc fitted in the suggestive formula mentioned above.

The separate first step was meant to "normalize" N quickly into the range between 0.1 and 1, before the real successive approximation started.

References:

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3. Rick Bensene, *Wang LOCI-2*, 1997-2010 http://www.oldcalculatormuseum.com/wangloci.html
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7. Wang, A., *Calculating Apparatus*, US patent 3,402,285, Filed Sept. 22, 1964; Granted Sept. 17, 1968 (The same patent has also been granted in Germany, UK, Netherlands,Sweden, and Switzerland.)
8. Advertisement for the LOCI-2, Scientific American, 215:3, Sept. 1966, p. 172