K&E 4102 Surveyor's Duplex Slide Rule

Richard Smith Hughes and Clark McCoy

Keuffel & Esser produced numerous specialized slide rules over the years: Chemist's Duplex (4160), RoyLance Electrical (4133, possibly the best of the Electro/Elektros), and the Kurtz Psychrometric (4175) to mention but three. The Surveyor's Duplex, 4102/N4102 (Figure 1), discussed here is a true masterpiece of slide rule design. We will discuss the historical background (Clark) first, followed by the slide rule's theory and operation (Richard).

Very little has been said about this elegantly complex slide rule produced by K&E from 1915 to 1949. A description of the rule is best done by quoting several paragraphs from the K&E catalogs.

"The fact that all astronomical data essential to surveying, such as azimuth, time, latitude, etc., can be ascertained by means of the usual type of Transit with vertical circle but without solar attachment, while is generally known, is rather seldom utilized in this country. The main reason for this surprising condition is the difficulty of computing, in the field, by spherical trigonometry, the results of the observations.

The K&E Surveyor's Slide Rule entirely eliminates this difficulty by reducing the hitherto complicated calculations to mere mechanical operations, thereby rendering the method of field astronomy with the regular Engineer's Transit extremely simple and practical."

The rule is constructed on a 20-inch duplex frame like the 4092-5. The 20-inch rule has enough resolution to match the accuracy of a standard transit that has a resolution of 1 minute of arc.

The front face of the rule has scales for determination of the meridian by direct solar observations. It also has the sine and cosine scales used for computing the latitudes and departures of the course.

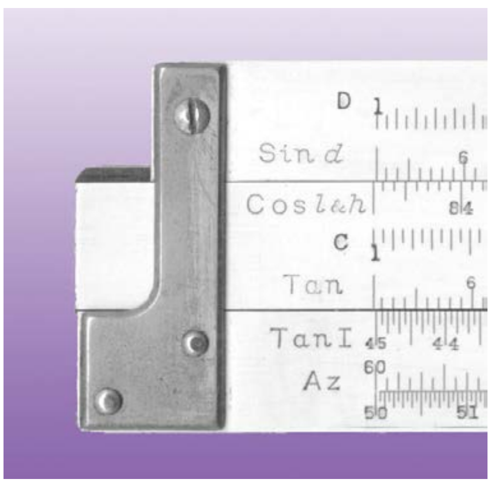


Figure 1

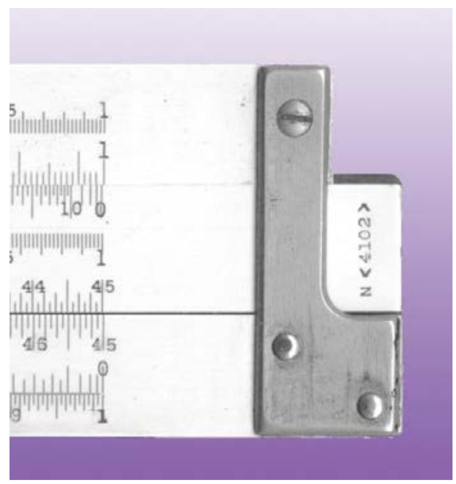


Figure 2

The back side of the rule has the normal scales as well as two full-length stadia scales.

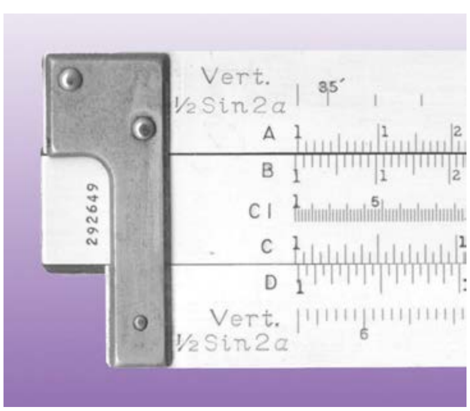


Figure 3

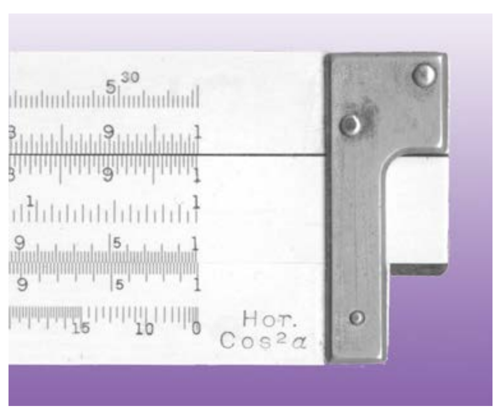


Figure 4

Table 1
E&E catalog entries for the 4102

Year	Model #	Cursor Type	Variants	Price
1915	4102	All glass, metal blocks	Probably not laminated edges	$18.00
1916	4102	All glass, celluloid blocks	Probably not laminated edges	$18.00
1921	4102	All glass	Probably not laminated edges	$26.50
1922	4102	All glass	Laminated edges & serial #	$26.50
1927	N4102/S	All glass	Only changed serial # location	$24.00/$25.50
1933	N4102/S	All glass with ridges		$24.00/$25.50
1936	N4102/S	Improved cursor		$24.00/$25.50
1943	N4102/S	Improved cursor	K&E logo on right front end	$28.00/$30.00
1949	N4102/S	Improved cursor	Last shown in catalog	Temp. Discontinued

The 4102 is first shown in the 1915 K&E full line catalog with an all-glass cursor for $18.00. $18.00 was a high price for that time period. It is not known at this time if this first issue had laminated edges on the rule. Most of the duplex rules of that time did not have laminated edges. Serial numbers did not appear on slide rules until 1922.

The above table shows the time line of variants of the 4102:

The model number change in 1927 to N4102 did not materially change the rule. The only observed changes in actual samples are the location of the serial number. It is believed that this was a production style change that affected most rules in the line at the time. The scales and their functions are identical between the 4102 and N4102. This change would have happened at about serial number 200000. David Ingram has provided scans of his 4102, serial number 104688, which dates around 1924-1925. This rule confirms that the only change between the 4102 and N4102 is serial number location.

All of the variants of the rule are the product of normal production changes that applied to the entire slide rule line of K&E. With very few exceptions, production changes were applied across the entire line at the same time.

The 4102 is a fairly rare rule. At this time I only know four collectors with them. I have seen about four or five for sale on eBay in the last ten years. The last one sold for $650 on eBay about a year ago. The N4102 that I have has a serial number of 292649 with an improved cursor. This rule probably dates about 1942. A friend bought it at a garage sale in Berkeley for $10. I paid a great deal more for it.

The 4102 is designed to solve for the Azimuth, Z, of the Sun from a known location:

$$Z = \cos^{-1}[(\sin d/(\cos h\cos l) - \tan h\tan l)]$$

where:

d = declination of the Sun (from Solar Ephemeris, accurate tables of solar and stellar locations at regular intervals of time, location, date, and time of measurement)

l = latitude of measurement at a known location (from map or measurement)

h = measured Sun's vertical angle at the known location

This equation was obtained from Johnson & Smith: *The Theory and Practice of Surveying*, 7*th* Ed., McGraw Hill, 1914, page 99. The slide rule is advertised as having an accuracy of 1' (which it does for several examples that were tried).

The four tables that follow describe the definitions, measurement procedures, N 4102 scales and limits (for determining the decimal point), and an example. The N 4102 removes the need for the Trig and Log tables necessary for determining Z and is a real time savor, especially in the field.

The backside of the N 4102 has the scales used in finding the horizontal distance (Hor. $\cos 2\alpha$) and vertical angle (Vert. (½)($\sin 2\alpha$)) in Stadia surveying (R. S. Hughes "Stadia or Tacheometric Slide Rules, Part II", JOS, 16:2, 2007).

The Sin *d*, Cos *l*&*h* and Az scales would also have been of use to those who needed accurate (within 1') Sin, Cos, and Tan values.

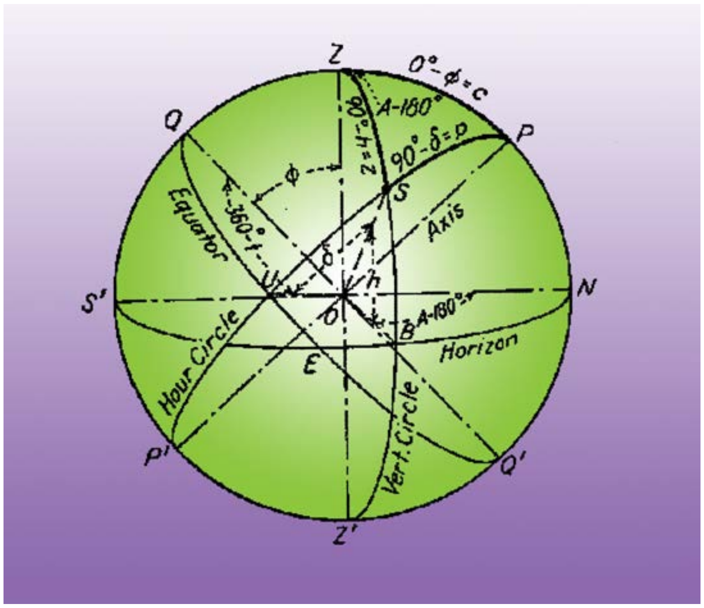
Special thanks go to Kenneth Kayser for tracking down and copying the instruction manual for the 4102, David Ingram for providing scans of his 4102, and Clay Castleberry for the picture of a 4102 case.

Table 2

Bearing Of The Sun

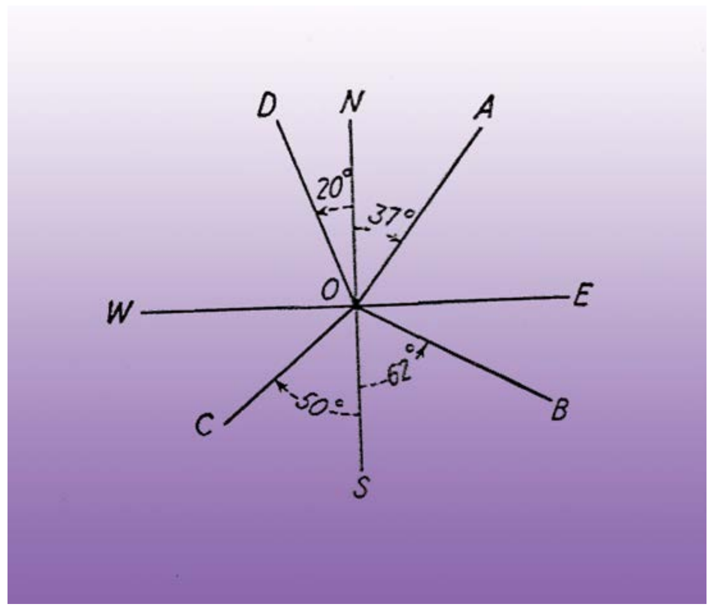
Spherical to Polar Coordinates

(Azimuth is classically measured from the north, however for azimuths, Z, greater than 90°, I use azimuths from the south. The Azimuth angle Z = Bearing angle)



Spherical PZS Triangle

Angle Z is spherical angle between the Sun, 0S, and True North, 0P, where 0 is the observers location



Polar coordinates and Bearing Quadrants

N is True North (0P from the figure at the left), 0A, 0B, 0C and 0D are the Sun's Bearing, Z, and 0 is the observers location

Sun's Location	Bearing	Quadrant	Comments
A	N37°E	I	A.M. measurements: $\sin d/(\cos h \cos l) > \tan h \tan l$: Z from the North
B	S62°E	IV	A.M. measurements: $\tan h \tan l) > \sin d/(\cos h \cos l)$: Z from the South
D	N20°W	II	P.M. measurements: $\sin d/(\cos h \cos l) > \tan h \tan l$: Z from the North
C	N50°W	III	P.M. measurements: $\tan h \tan l) > \sin d/(\cos h \cos l)$: Z from the North

Azimuth angle, $Z = \cos^{-1}[(\sin d/(\cos h \cos l) - \tan h \tan l)]$ = Bearing angle

where:

d = Declination of the Sun (from Solar Ephemeris at the location, date, and time of measurement); l = latitude of measurement location (from map or measurement); h = measured Sun's vertical angle

Table 3

Measurement Procedure For Bearing Of Location B
(The optimum times for measurements are: 8 AM to 10 AM and 2 PM to 4 PM)

•Place Transit at known location, A: the Latitude at A is known from a map or measurement:

•Zero the horizontal scales, $0^o 00.0'$ and rotate Transit to location B and lock the lower horizontal plate.

•Unlock the horizontal vernier plate, rotate the Transit and measure the vertical angle of the sun, h.
The declination, d, is known from the Solar Ephemeris for the date and time of measurement.
Read the horizon angle from location B to the Sun, θ, on the horizontal angle scales.

•Calculate the Sun's bearing, Z, from location A (see Table 2)
$Z = \cos^{-1}[(\sin d/(\cos h \cos l) - \tan h \tan l)]$ or $\cos^{-1}[(\tan h \tan l) - \sin d/(\cos h \cos l)]$

•Calculate the Bearing of location B
Bearing B = Z – θ

Table 4

N 4102 Scales
(For decimal point determination)

Sin*d* referenced to the D scale: two scales
Left end: $5^o 44.35'$ (Sin*d* = 0.1, the left D Index) to 24^o (Sin*d* = 0.4067, on the D scale)
Right end: $2^o 22'$ (sin*d* = 0.0413) to $5^o 44.35'$ (sin*d* = 0.1, the right D scale Index)
Cos*l*&*h* referenced to the C scale
$84^o 15.65'$ (Cos*l*&*h* = 0.1, the left C Index) to 0^o (Cos*l*&*h* = 1, the right D Index)
Tan referenced to the C scale
$5^o 42.6'$ (Tan = 0.1, the left C Index) to 45^o (Tan = 1, the right C Index)
Tan I (inverted tan scale (1/tan) referenced to the D scale): Two scales
Left end: 45^o (1/Tan = 1, the left D Index) to $20^o 50'$ (1/Tan = 2.628 on the D scale)
Right end: 75^o (1/Tan = 0.268 on the D scale) to 45^o (1/Tan = 1, the right D Index)
Az: two scales, bottom = cosZ, and top $\cos^{-1}$ = Z
Bottom scale (cosZ): 0.5 (the left D Index) to 1 (the right D Index)
Top scale, Az ($\cos^{-1}$): $60^o(\cos^{-1} = 0.5)$ to $0^o(\cos^{-1} = 1)$
If $\cos^{-1}Z<0.1$ ($Z > 84^o 15.65'$) use the right Sin*d* and D scales
$Z = 90^o - \sin^{-1}(\cos Z)$

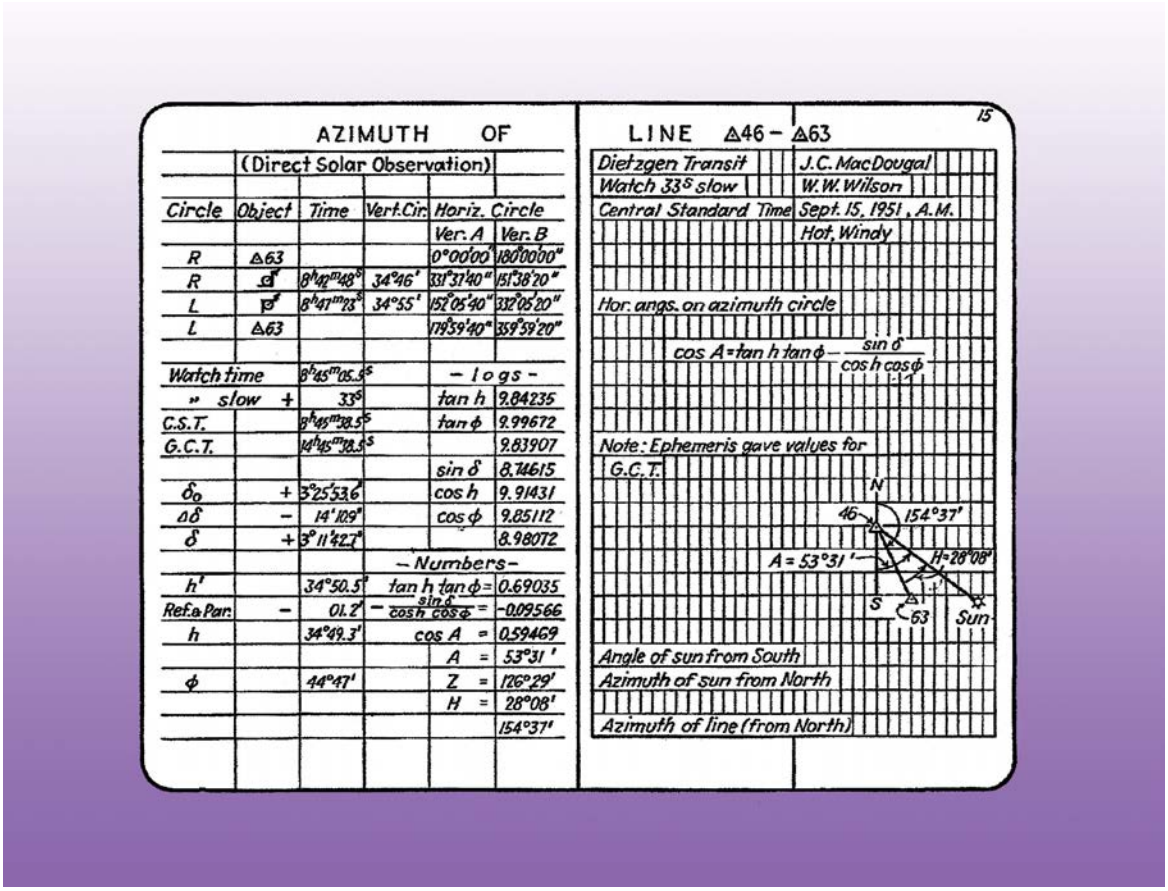
The scale values are used in determining the sind/(cosh cosl) and tanhtanl decimal points.

Table 5

Example
From Davis and Foote *Surveying*, 4th ed. McGraw-Hill, 1953

Given
Time of measurement: 15 Sept, 1951, 8:45 AM (Quadrant I or IV-see Table 1)
d (from 1951 Solar Ephemeris) = $3^o 11.71'$ l = $44^o 47'$ h = $34^o 49.3'$ θ = $28^o 08'$ (see Table 2)

Surveying notes for the example: Davis and Foote, page 527



AZIMUTH OF LINE Δ46 – Δ63

	(Direct Solar Observation)				
Circle	Object	Time	Vert.Cir.	Horiz. Circle Ver. A	Ver. B
R	Δ63			0°00'00"	180°00'00"
R	☉	$8^h42^m48^s$	34°46'	331°37'40"	151°38'20"
L	☉	$8^h47^m23^s$	34°55'	152°05'40"	332°05'20"
L	Δ63			179°59'40"	359°59'20"
Watch time		$8^h45^m05.5^s$		– logs –	
" slow	+	33^s		tan h	9.84235
C.S.T.		$8^h45^m38.5^s$		tan φ	9.99672
G.C.T.		$14^h45^m38.5^s$			9.83907
				sin δ	8.74615
δ_0	+	3°25'53.6"		cos h	9.91431
Δδ	–	14'10.9"		cos φ	9.85112
δ	+	3°11'42.7"			8.98072
			– Numbers –		
h'		34°50.5'		$\tan h \tan \phi =$	0.69035
Ref.& Par.	–	01.2'		$-\frac{\sin \delta}{\cos h \cos \phi} =$	–0.09566
h		34°49.3'		cos A =	0.59469
				A =	53°31'
φ		44°47'		Z =	126°29'
				H =	28°08'
					154°37'

15

Dietzgen Transit | J.C. MacDougal
Watch 33^s slow | W.W. Wilson
Central Standard Time | Sept. 15, 1951, A.M.
Hot, Windy

Hor. angs. on azimuth circle

$$\cos A = \tan h \tan \phi - \frac{\sin \delta}{\cos h \cos \phi}$$

Note: Ephemeris gave values for G.C.T.

Angle of sun from South
Azimuth of sun from North
Azimuth of line (from North)

Note

δ (our d) = 3°11'2.7" = 3°11'.42.7'

Δ46 is our location A, Δ63 is our location B, Φ is our latitude l (44°47'), H is our θ (28°08').

The Bearing of the Sun, A (our Z) = S53°31'E

The Bearing of Δ63 (our B) = A - H (our Z - θ) = S25°23'E

Exact Solution (hp 15C)

sin*d*/(cos*h*cos*l*) = 0.09866 tan*h*tan*l* = 0.69034

tan*h*tan*l* > sin*d*/(cos*h*cos*l*) Z measurements from the South, Quadrant IV (Table 1)

tan*h*tan*l* - sin*d*/(cos*h*cos*l*) = 0.59468

Z (Bearing of Sun) = S53°30.6'E

Bearing of B = Z – θ = S25°22.6'E

N 4102

Using the Sin*d* and Cos*l*&*h* scales

sin*d*/(cos*h*cos*l*) = {[3°11.71' on the Sin*d* scale]/[(34°49.3' on the Cos*l*&*h* scale)]}{[1/(34°49.3' on Cos*l*&*h*} = 0.957 on the D scale

Using the Tan and Tan I scales

tan*h*tan*l* = [34°49.3' on the left Tan I scale]/ [44°47' on the Tan scale] = 0.69 on the C scale below the D Index

tan*h*tan*l*>sin*d*/(cos*h*cos*l*) Quadrant IV (see Table 1)

tan*h*tan*l* - sin*d*/(cos*h*cos*l*) = 0.5943 (use Cos*l*&*h* and C scales to find Z)

0.5943 on the C scale = 53°30.1' on the Cos*l*&*h* scale

Z = S53°30.1'E (Exact S53°30.6'E)

Bearing of Location B = Z – θ = 53°30.1' - 28°08' = S25°22.1'E (Exact S25°22.6'E)

The calculated results are within the 1' advertised slide rule accuracy

Result Comparison

Bearing of:	Exact	Davis & Foote	4102
Sun, Z, at location A	S53°30.6'E	S53°31'E	S53°30.1'E
Location B	S25°22.6'E	S25°23'E	S25°22.1'E

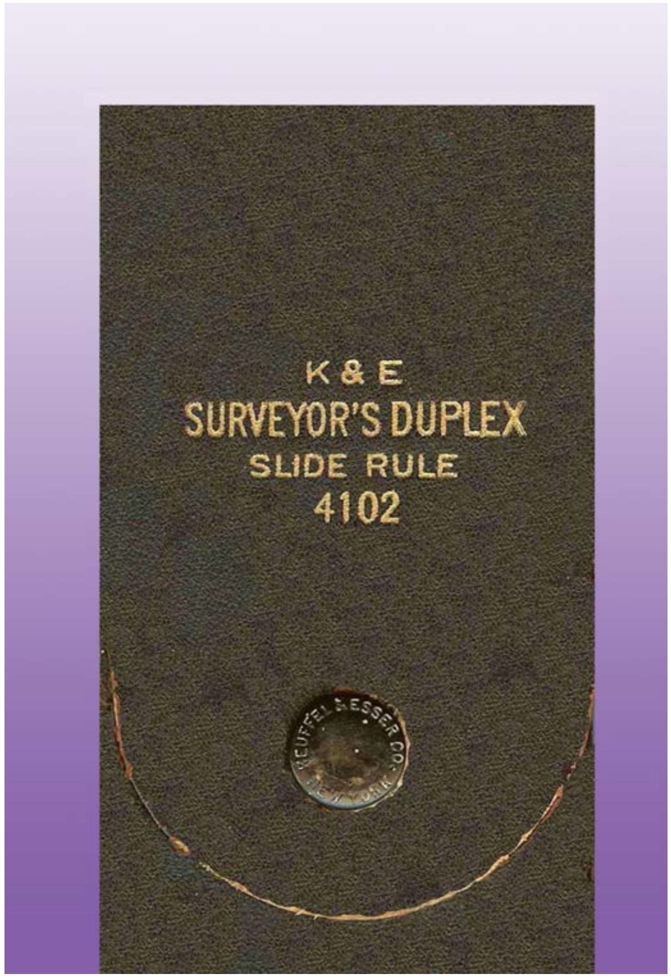


Figure 5.
The case for Clay Castleberry's N4102

Richard Smith Hughes is a retired electronic engineer, currently a researcher of slide rules, quantum mechanics, and the Textual and Typological evolution of Ancient Egyptian Middle Kingdom coffins.

Clark McCoy is a retired engineer from the broadcast and tape machine industries. He mostly collects K&E slide rules, planimeters, and K&E literature. His extensive collection of K&E catalogs is the foundation for his K&E catalog website.