Slide Rules, Sextants, and the Sky

Instrumentation Milestones In Aerial Celestial Navigation

Paul McConnell

Foreword

This paper is an expansion of an outline created for distribution at the Oughtred Society's 2005 Fall Meeting, held at the MIT Museum in Cambridge, Massachusetts. The new material includes a brief sketch of spherical trigonometry as applied to celestial navigation, a few diagrams, four examples of the solution of the Navigational Triangle using different methods of computation, and descriptions of a few additional slide rules and instruments for aviators. The Bygrave Position Line Slide Rule remains the central subject.

Background: Marine Navigation

Piloting, **dead reckoning**, and **celestial navigation** are the three traditional methods of directing the course of a ship. These methods are usually associated with the navigation of boats and ocean-going vessels, and evolved from knowledge gained over centuries of human exploration and travel.

Piloting, certainly the earliest form of navigation, uses distinctive landmarks for points of reference. Near the shore the pilot's knowledge of depths, currents, and tides is essential. Farther off the coast the pilot needs charts and instruments to navigate. The pilot's instruments may include drafting instruments, a magnetic compass, a lead and line or sounding machine, a pelorus, and a stadimeter. Maritime authorities often install artificial aids to navigation, such as buoys and lighthouses, to assist the pilot.

Dead reckoning relies on estimates of direction and distance traveled to predict a vessel's current position relative to its last known location (hence deductive reckoning). Useful in bad weather or beyond the sight of land, dead reckoning requires charts, drafting instruments, a magnetic compass, a reliable timekeeper, and a patent log to measure distance traveled. Dead reckoning provides only an estimate of position, so other methods of navigation must be used whenever possible to correct the inevitable errors.

Celestial navigation determines an observer's position by reference to astronomical objects. Even the earliest mariners derived useful information from the heavens. The North Star, Polaris, for example, was certainly a useful point of reference. By the end of the 19^{th} century, progress in science, continuing refinements in methods, and the development of improved instruments, made celestial navigation a key element in safe transoceanic navigation.

An Overview of Celestial Navigation

The celestial navigator employs the Sun, the Moon, and selected planets and stars as points of reference for determining the ship's location on the surface of the Earth. For practical purposes, the Earth is taken to be a sphere rotating at the center of a larger sphere of infinite radius. The larger Celestial Sphere is imagined to be dotted with the celestial bodies. To an observer on Earth the stars appear to remain fixed in relation to one another, while the Sun, the Moon, and the planets appear to wander about with the passage of time. Learning to accurately predict the wanderings of the astronomical bodies was a significant challenge for early astronomers.

Astronomers had created tables for predicting the position of objects on the Celestial Sphere as early as the second century. It was not until the 17^{th} century, however, that astronomers developed mathematical models sufficiently accurate to offer the reliability desired by European navigators. Logarithms and movable-type printing enabled the economical computation and publication of astronomical tables for navigators. The earliest British astronomical almanacs date to the 1760s.

By the end of the 20^{th} century, a publisher, usually under the auspices of a national government, used current mathematical models to compute astronomical tables for navigators. The *Nautical Almanac*, for example, was a cooperative effort of The Royal Greenwich Observatory and the U.S. Naval Observatory, published annually by Her Majesty's Stationery Office and the U.S. Government Printing Office. The tables in the *Nautical Almanac* list the positions of selected astronomical objects in relation to a celestial reference named Aries, the point on the Celestial Sphere that marks the Vernal Equinox.

Navigators record and update the position of their vessels on charts specific to the surface of the Earth. Those positions are expressed in relation to Earthly landmarks such as the poles of the Earth's axis of rotation, the Equator, or the Greenwich Meridian. The celestial landmarks listed in the astronomical tables, however, continuously change their position relative to the navigator's Earthly landmarks. According to one source, "The principal problem of celestial navigation is that of converting the coordinates of one system to those of another[1].

The dominant factor in the relation between the Earthly coordinate system and the celestial coordinate system is the passage of time. The Earth's daily rotation on its axis, and its annual revolution around the sun, set the pace for the changes observed in the heavens. Entries in the *Nautical Almanac* are arranged by date and time, and the ability to measure time is essential to successful celestial navigation.

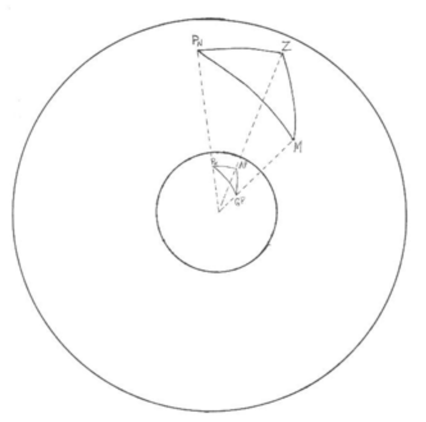

Figure 1. A sphere within a sphere.

[1] From *Navigation and Nautical Astronomy*, 1951.

Referring to Figure 1, one sees the Earth within the imaginary Celestial Sphere. On the image of the Earth there is drawn a Navigational Triangle formed by the three points:

PN – North Pole

AP – Assumed Position of the vessel

GP – Geographical Position (or sub-point) of the astronomical body.

The Navigational Triangle is also projected onto the Celestial Sphere, where the three points are labeled:

PN – The Celestial North Pole

Z – The observer's Zenith

M – The point on the Celestial Meridian where the body appears.

The spherical surfaces on which the triangles are projected share a common center. The triangles are proportional. The arc forming each side of one triangle subtends the same angle as the arc of the corresponding side of the other triangle. Given the proper instruments, a navigator may measure some of those angles. Spherical trigonometry then offers the means to calculate the unknown quantities from the known values.

In spherical trigonometry, the sides of a triangle are most often measured in degrees of arc instead of units of length. When it is necessary to measure distances in length, navigators use the nautical mile. The nautical mile is defined as the length of one minute of arc of a great circle of the Earth, or approximately 6080 feet.

The Celestial Navigator's Tools

From the overview above, one may establish a set of prerequisites for performing celestial navigation. In addition to the charts and instruments used for dead reckoning, a celestial navigator must have access to:

- A means of predicting the positions of the celestial bodies for a given instant in time
- An instrument to accurately indicate the current time
- A method of computing the position of a body in the necessary frame of reference
- An instrument to observe the body and measure its position relative to the Earth.

The astronomical almanac satisfied the first requirement. John Harrison's perfection of the marine chronometer in 1735 answered the need for a timekeeper. The mathematics of spherical trigonometry offered a method for computing the relative positions of the reference points. Finally, the marine sextant, developed by both Hadley and Godfrey in 1730, served as the observation and measuring instrument.

With the proper tools at hand, mariners could employ celestial techniques more frequently, and with increasing reliability. As proficiency improved, so did the process. In 1837 Captain Thomas Sumner, a merchant seaman from the United States of America, introduced a significant improvement in navigation techniques with his Celestial Line of Position. In 1874 Admiral Marcq Saint Hilaire of the French Navy developed his Intercept Method for determining the Celestial Line of Position. Saint Hilaire's contribution simplified celestial navigation and became a favored method of determining a ship's position.

The Saint Hilaire Intercept Method

Using the Saint Hilaire Intercept Method, a navigator first selects an opportune celestial body, a pre-determined position, and a time for taking a sextant observation. The predetermined position is where, based on dead reckoning navigation, the observer expects to be when the sighting is to be taken. This position is called the Assumed Position (AP). Next, the navigator computes the solution to the Navigational Triangle for the selected AP and time. The solution will give the navigator the altitude (elevation above the horizon) and azimuth from the AP to the astronomical body at the instant in time when the observation is to be taken.

At the appointed time, the navigator measures the altitude of the body with the sextant. The difference between the computed altitude and the observed altitude will indicate the distance from the AP to the observer's Line of Position (LOP). The azimuth shows the direction along which the difference lies.

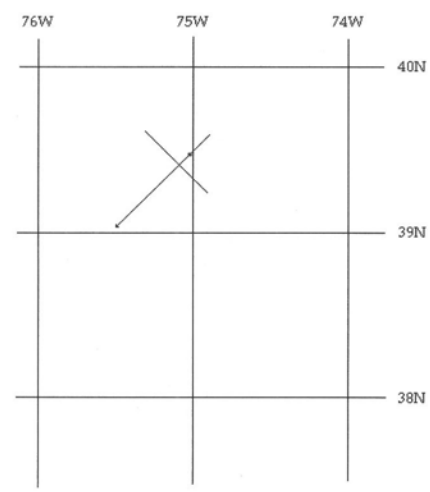

Figure 2. Chart grid with Ap, Azimuth and LOP.

In the figure above, the AP is shown as a dot on a chart at approximately 75 degrees 2 minutes West Longitude and 39 degrees 29 minutes North Latitude. The navigator will look in the *Nautical Almanac* for the body to be observed, using the date and time of day the observation will be taken. Using one of several popular methods, the navigator will compute the altitude and azimuth from the AP to the selected body. Then, at the chosen time, the navigator will measure the observed altitude of the body. In the figure, the line running southwest from the AP is the azimuth to the body. The line drawn perpendicular to the azimuth is the LOP. The LOP is *toward* the body if the observed altitude is *greater*, and *away* from the body if the observed altitude is *less* than the computed altitude. In the example, the observed altitude was approximately 5.5 minutes greater than the computed altitude, so the LOP is 5.5 nautical miles closer to the body than the AP. A second LOP, taken from a different celestial body, would place the observer at the intersection of the two LOPs. Such a determination is called a fix.

Basic Formulae for Spherical Trigonometry

The fundamental formula of spherical trigonometry, the Cosine Formula, is shown below.

For the spherical triangle:

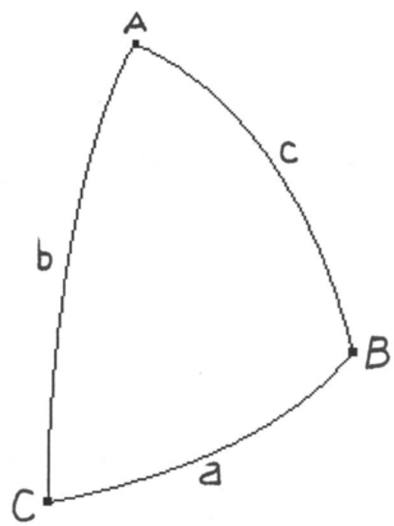

Figure 3. Spherical Triangle A B C.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

The Cosine Formula includes all three sides of the triangle and one angle. If two sides and one angle are known, one can solve for the third side. This formula may be applied to the Navigational Triangle to solve for the unknown side.

To simplify their calculations, navigators have employed several variations of the Cosine Formula. One popular rearrangement was the Cosine-Haversine Formula, shown for solving the Navigational Triangle in the next figure.

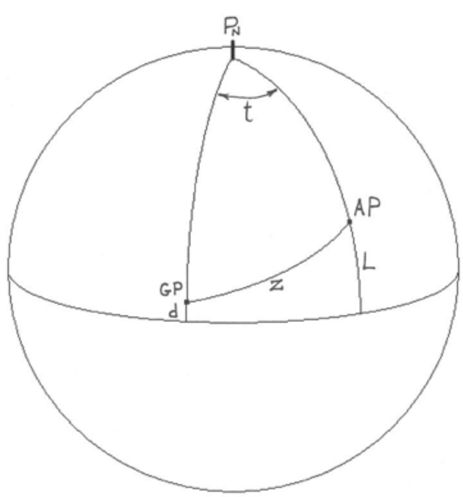

Figure 4. Spherical Triangle A B C.

$$hav z = hav\ (L \sim d) + \cos L \cos d\ hav\ t$$

where

z is the Zenith Distance to the body
L is the latitude of the AP
d is the declination of the body
t is the Meridian Angle to the body.

The expression $(L \sim d)$ is interpreted as $L - d$ if the latitude and declination have the same name (both North or both South); $L + d$ if the latitude and declination have the opposite names (one North, one South).
This formula was popular because it used only two functions: cosine and haversine. It was used with tables of the trigonometric functions and their logarithms.

Having solved for side **z**, the navigator can easily solve for the computed altitude. Imagine viewing the Earth along the axis of the great circle of which the arc **z** is a part, as shown in the following figure.

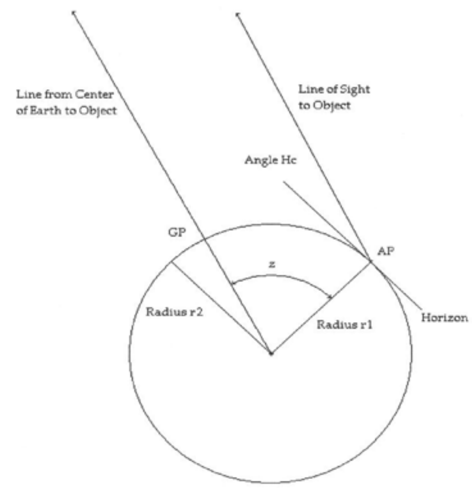

Figure 5. Derivation of altitude from zenith distance.

The line from the center of the Earth to the body and the line of sight from the AP to the body are taken to be parallel. The angle between radii **r1** and **r2** is a right angle in the plane of the great circle. The horizon at the AP is tangential to the Earth, so it is perpendicular to radius **r1** and parallel to radius **r2**. The complement of the angle representing side **z** is equal to the angle between the horizon and the line of sight from the AP to the body, the Computed Altitude (Hc).

Computed Altitude (Hc)= 90 – z

(See **Appendix A** for an example of solving for Computed Altitude using the Cosine-Haversine Formula.)

The Transition From Sea to Air

The preceding background information sketches the state of celestial navigation practice that was the foundation upon which aerial navigators built. The following sections chronicle the development of celestial navigation instruments, especially those designed primarily for use on aircraft. The focus will be on:

- Instruments for observing and measuring the altitude of celestial objects,
- Tables and instruments used for computing the solution to the Navigational Triangle.

Some of the instruments described were found only on combat aircraft. This is because extraordinary demands are routinely asked of combat crews and their aircraft. Long flights over the oceans, flights through dangerous weather conditions, flights over hostile territory, the need for radio silence, the effects of battle damage, and the requirements of automated weapon systems, all suggest the need for robust equipment, redundant systems, and alternate sources of navigation data.

Airship Navigation

A sextant measures the angular distance from the horizon to a celestial object. Because the natural horizon could be obscured by weather conditions or dark of night, some 19^{th} century marine navigators searched for an alternate reference. Several forms of artificial horizon were proposed, including one formed in a bowl of mercury, a horizontal reference stabilized by a pendulum, a gyroscopic horizon, and a bubble horizon similar to a carpenter's spirit level. Admiral Fleuriais of France proposed a gyroscopic horizon as early as 1885. The Parisian firm of Ponthus et Therrode obtained a U.S. patent on Fleuriais' gyroscopic sextant in 1902. Salomon August Andree developed a bubble sextant in 1897.

When aviators began to experiment with the navigation of airships in the 19^{th} century they borrowed from the vast store of knowledge accumulated by sea-going navigators. Most of the technology used for marine navigation could be applied directly to aerial navigation. One notable exception was the nautical sextant. When taking a sighting from high above the Earth, the horizon was no longer a useful reference for measuring the altitude of an astronomical object. Aerial navigators joined the search for an alternative to the natural horizon.

The Marcuse sextant, manufactured by the Butenschon firm of Hamburg in 1901, was the first successful bubble sextant sold for aerial use. Pendulum-stabilized horizons included the French Fave design of about 1901; a design by Ernst G. Fischer of the U.S. Coast & Geodetic Survey; and a sextant constructed by Keuffel & Esser for ". . . the celebrated attempt of Wellman and Vaniman to cross the Atlantic Ocean with the dirigible *America* in 1910. On this voyage a sextant with an artificial horizon was used for the absolute determination of position . . ."[2].

The Schwarzschild bubble sextant for balloons and airships was introduced in 1913.

Incorporating several refinements over its predecessors, the Schwarzschild established the bubble's superiority over the other forms of artificial horizon.

Several of the sextants of this period included a battery-operated electric lamp to illuminate the bubble for observations at night.

Slide Rules for Airships

Aside from celestial navigation, airship crews had another responsibility that required on-board computation: gas volume management. Changes in altitude, barometric pressure, and air temperature, all affected the volume of the lifting gas. The changes in gas volume caused changes in lift, and could lead to venting of the buoyant gas. Maintaining the correct lift and avoiding wasteful loss of the lifting gas were important tasks. Consequently, there was a need for solving gas volume problems while aloft.

Several specialized slide rules were designed to meet this need. A few of those rules were described in the 1926 book *Aircraft Instruments*, by members of the staff of the Aeronautical Instrument Section of the U.S. Bureau of Standards. One of the slide rules was manufactured by the Albert Nestler firm, and was ". . . very complicated and has 10 scales and 3 slides"[3]. A simpler British slide rule with four scales, the Scott-Teed rule, was also mentioned. Finally, the book described a modified version of the Scott-Teed rule, designed by the Gas-Chemistry Section of the U.S. Bureau of Standards and manufactured by the Eugene Dietzgen Company.

The Bureau of Standards rule was based on 20-inch scales. It had two side rails, one slide, and two runners, one of which could be clamped in place. An index mark "V" was inscribed onto the C and D scales to show the volume in cubic feet of the particular airship on which the rule was used. The scales were arranged as follows:

A: 20 to 32 inches of mercury - logarithmic Barometer scale, on the top rail.

B: 0 to 25000 feet – linear altitude scale at the top of the slide.

E: 75 to 100% logarithmic Gas Purity scale,	F: -50 to +150 deg F logarithmic Temperature scale for volume calculations,	K: -50 to +150 deg F logarithmic Temperature scale for lift calculations,

(scales E, F, and K are spaced across the middle of the slide).

C: 10 to 100 - logarithmic scale at the bottom of the slide.

D: 10 to 100 - logarithmic scale on the bottom rail.

The airship slide rule was designed for use with hydrogen gas. The designers suggested that equivalent scales for helium could be added to the reverse side of the rule.

Airplane Navigation

Long-range navigation was more difficult for heavier-than-air craft. Charts and instruments were difficult to manage in a windy open cockpit, especially for a lone pilot. The airplane's pitching and rolling caused errors in magnetic compass indications. The wind caused difficulties in measuring ground speed and drift, which led to dead reckoning errors. Accelerations and turbulence affected the stability of artificial horizons for sextants.

The difficulties aviators experienced in navigating their aircraft during World War I prompted a serious study of aerial navigation techniques and instruments by the warring nations. At Langley Field in 1919 H.N. Russell experimented with a glycerin-damped pendulum horizon. The device was attached to the aircraft and used with a marine sextant. Prof. Robert Wheeler Willson of Harvard University and Lt. Cmdr. Richard E. Byrd of the U.S. Navy both produced bubble attachments to adapt marine sextants for aerial use, also about 1919.

In England at about the same time Lionel Barton Booth

[2]From *NACA Technical Report 131 – Aerial Navigation and Navigating Instruments* by H.N. Eaton, U.S. Bureau of Standards, 1923.

[3]From *NACA Technical Report 160 – An Airship Slide Rule* by E.R. Weaver and S.F. Pickering, U.S. Bureau of Standards, 1923.

developed a bubble sextant that differed significantly from the traditional marine sextant. This device became the basis for a series of bubble sextants developed by the Royal Aircraft Establishment (RAE) at Farnborough.

Methods for Performing Calculations Aboard Airplanes

The RAE bubble sextant offered aviators an instrument for taking celestial sightings in flight, but there remained a need for a simple, compact means for solving the Navigational Triangle while airborne. Once again, aviators borrowed from their nautical counterparts. Methods they tried include computation using trigonometric formulae, nomograms, graphical methods, tables of pre-computed solutions, and mechanical calculating devices.

The Cosine-Haversine formula remained popular, but it was complicated and too slow for use on airplanes. Several graphical systems and nomograms appeared, but did not receive wide acceptance at this time.

For tabular solutions, U.S. aviators relied on the proven publications of the U.S. Navy. In addition to the Naval Observatory's *Nautical Almanac*, aviators adopted Hydrographic Office Publication Number 201 (H.O. 201), *Simultaneous Altitudes and Azimuths of Celestial Bodies*, and H.O. 203, *The Sumner Line of Position*. Other well-known tables, such as those by Commander DeAquino of the Brazilian Navy and Doctor S. Ogura of the Hydrographic Department of the Imperial Japanese Navy, were also tried. H.O. 208, *Navigation Tables for Mariners and Aviators*, by J.Y. Dreisonstok, and H.O. 211, *Dead Reckoning Altitude and Azimuth Tables*, by A.A. Ageton, appeared by 1931.

In the mechanical arena, several devices for solving the navigational triangle were introduced, but most did not succeed. One instrument, however, did show promise as an ideal navigation computer: the slide rule.

Although not designed for the celestial navigator, one slide rule for military aviators was suggested in March, 1918. An article in *Aerial Age Weekly* proposed a slide rule for solving aerial bombing problems. The author described a simple 8-inch slide rule with three scales on each side. The front of the rule bore an A scale of degrees, a B scale of height in feet, and a C scale of miles per hour. These scales solved for the bombing angle. The back of the rule bore an A scale of mph, a B scale of seconds, and a C scale of height in feet, and solved for ground speed. There is no evidence the bombing slide rule saw actual use in combat.

In 1920 Dr. Charles Lane Poor received a British patent for his Line-of-Position Computer. Dr. Poor (1866 - 1951) was a professor of astronomy at Columbia University, and a prominent yachtsman. He published several books on astronomy and navigation, and disputed Einstein's Theory of Relativity before it gained wide acceptance.

Dr. Poor's Line-of-Position Computer solved celestial navigation problems via the Cosine-Haversine Formula. The instrument was a massive circular slide rule, 15 inches in diameter and 4.7 pounds in weight. It had 8 concentric scales of trigonometric functions, a transparent disk with an index line, and a movable arm with a locking mechanism. Although an impressive artifact—the Nystrom's Calculator of Celestial Navigation, if you will—Poor's Line-of-Position Computer was too bulky and complex for practical aerial use.

The Air Ministry Position Line Slide Rule

A more successful celestial navigation slide rule, introduced at about the same time as Poor's, was the Air Ministry's Position Line Slide Rule. This slide rule was developed by Captain Leonard Charles Bygrave at the Air Ministry Laboratory, South Kensington. Bygrave was a creative and original engineer. Before serving in the RAF, he was employed as a telephone engineer by the Relay Automatic Telephone Company. He received several patents for automatic telephone systems from 1915 through 1917. In 1919 he received a patent for a unique bombsight design. In later years he received several more patents for instruments relating to aircraft navigation.

Bygrave's Position Line Slide Rule was designed specifically for use in aircraft. It solved the Navigational Triangle defined by one of the Earth's poles, the Geographical Position (GP) of the object under observation, and the Assumed Position (AP) of the observer, as used in St. Hillaire's Intercept Method. Instead of the popular Cosine-Haversine formula, Bygrave chose a Tangent-Cosine relationship.

If the celestial triangle is divided into two right triangles by dropping a perpendicular line from the observer's meridian to the GP, the following formulae apply:

$$\tan y = \tan d / \cos t$$

If L and d are the same name (both North or both South), $Y = (90 - L) + y$

If L and d are the opposite name (one North, one South), $Y = (90 - L) - y$

$$\tan A = \cos y \tan t / \cos Y$$

$$\tan h = \cos A \tan Y$$

where h = altitude, L = latitude, d = declination t = meridian angle (similar to hour angle), A = azimuth, Y & y are intermediate values.

These formulae use only two functions: tangent and cosine. Bygrave's slide rule incorporated helical scales spiraled around two concentric cylinders. The inner cylinder was inscribed with logarithms of tangents and the outer cylinder was inscribed with logarithms of cosines. A third cylinder surrounding the two graduated tubes served as the cursor, and was imprinted with an abbreviated set of instructions.

By carefully selecting the optimum formula and scale layout, Bygrave reduced the solution of the Navigational Triangle to a dozen moves. An experienced navigator could use the instrument to obtain a solution in two or three minutes. The long, spiral scales provided an accuracy of one minute of arc. The hollow cylinders weighed little, and the telescoping design occupied minimal space when not in use. Bygrave's careful attention to work flow and mechanical design resulted in an instrument well adapted to solving the Navigational Triangle while in flight. The A.M. Position Line Slide Rule, along with the RAE bubble sextant, became the standard celestial navigation instruments for the RAF. Henry Hughes and Son made the slide rule, and it remained popular with both aviators and marine navigators through the 1930s. Philip Van Horn Weems, a 1912 graduate of the U.S. Naval Academy and a dynamic proponent of aerial navigation, said Bygrave's Position Line Slide Rule " . . . is probably the most convenient mechanical computer for obtaining position lines from sextant

[4]From *Air Navigation* by P.V.H. Weems, McGraw-Hill, New York, 1938 (Second Edition).

observations."[4] Dennert and Pape produced a German language version, and there was also a Japanese language version, both probably dating from the 1940s.

(See **Appendix B** for an example of solving the Navigational Triangle with the Bygrave Position Line Slide Rule.)

Aerial Celestial Navigation Matures

Civil aviation advanced right along with military aviation. The U.S. Post Office Department experimented with aerial mail delivery as early as 1911. In cooperation with the Army Signal Corps Air Service, the Post Office Department inaugurated official airmail service on 15 May 1918. During the next decade the Post Office Department expanded its airmail operations from an initial Washington–New York daylight route to a nationwide network of around-the-clock airways, including air fields, beacons, and radio communication equipment. When commercial contractors took over the last of the Post Office Department airmail routes in late 1927, the stage was set for modern commercial air transportation. Outside of the research community, however, most practicing aviators paid little attention to celestial navigation. Piloting and dead reckoning reigned, since most early airmail and airline routes were over land.

The first scheduled trans-Atlantic air service began with the crossings of the dirigible *Hindenburg* in 1936. Long commercial flights over water were also being introduced by airlines in the mid-1930s. Over the ocean there were no landmarks for piloting. Even radio beacons were useless far from shore. Aviators took a renewed interest in celestial navigation.

Flight crews grew to include a dedicated navigator, and aircraft manufacturers made room for a chart table and an astrodome. The astrodome was an optically uniform transparent dome beneath which the navigator had a clear view of the heavens and protection from the wind.

The renewed attention to celestial navigation stimulated researchers and manufacturers to develop new, more capable instruments. At the United States Bureau of Standards, Dr. F.L. Hunt and Karl Hilding Beij developed an aircraft sextant that was subsequently manufactured by the Bausch and Lomb Optical Company. The Pioneer Instrument Company produced its aircraft octant. Both of these instruments employed the bubble form of artificial horizon.

In England, Hughes produced the Booth/RAE sextant. C. Plath of Hamburg, Germany, made a sextant with an artificial horizon designed by Admiral Coutinho of the Portuguese Navy. The Coutinho sextant resembled a marine sextant, and used a pair of spirit levels for its horizon. Captain Anton Wittemann used Plath's Coutinho Sextant when he served as navigator on the *Graf Zeppelin.* Wittemann later survived the fiery crash of the *Hindenburg.*

In 1935 Philip Van Horn Weems conducted a study of bubble sextant observations. Weems made 110 observations of the sun during a 700-mile flight in an open cockpit airplane. The observations were taken in eleven sets of ten, and the location where each set was taken was verified by reference to visual landmarks. Analysis of the data confirmed that averaging the results of several sextant observations tended to nullify the errors caused by the erratic meanderings of the bubble in its chamber, the bubble sextant's flaw.

By 1936 Philip Francis Everitt at Hughes and Son in England, the Kollsman Instrument Company of New York (founded by Paul Kollsman, inventor of the sensitive altimeter), and Lt. Thomas L. Thurlow of the U.S. Army Air Forces, were all trying to perfect a sextant averager.

Sextant averaging devices appeared in three forms:

- **Mechanical averagers** that summed fractions of several readings to form an arithmetical average,
- **Median recorders** that created a graphical record of the several readings, from which the navigator could estimate the median value of all of the individual readings,
- **Mechanical integrators** that continuously kept a running average of the altitude value over a period of time, typically a few minutes.

The World War II Era

P.F. Everitt's mechanical averager was offered commercially by Hughes and Son in 1938. A later version, the Mark IXA averaging sextant, ran for two minutes during which time it registered a measurement every two seconds, adding 1/60 of the value of each successive observation to a running total. A shutter fell to block the eyepiece after a full run of two minutes, when the average of all sixty measurements could be read from the instrument.

In 1938, the concept of a median recorder appeared from several sources. The median recorder was simpler to manufacture and simpler to use than a mechanical averager. In use, the navigator pressed a button each time an observation was to be taken. Pressing the button caused a stylus to inscribe a mark on a wheel or disk. After taking the necessary number of observations the navigator could visually determine the median of all of the marks on the recording surface. The navigator then aligned the median under an index mark and read off its numerical value from a graduated scale.

Several U.S. instrument makers supplied median recording sextants to the USAAF. The Pioneer Instrument Company made the A-7 aircraft sextant, based on a design by Victor Carbonara. The Bausch & Lomb Optical Company produced the AN-5854-1 aircraft sextant. The Fairchild Camera and Instrument Corporation made the A-10 sextant, a development of Thurlow's design, and Link Aviation Devices, Inc. made the A-12 sextant.

Harold E. Gray of Texas, working with Hughes in England, and Richard Francis Deimel and William Alexander Black, working with General Time Corporation in the U.S., ultimately received patents on mechanical integrators for use as sextant averagers. By the war's end Hughes had incorporated Gray's integrator into their aircraft sextants.

The installation of automatic pilot equipment on long range aircraft also helped reduce the errors from bubble horizons. The automatic pilot improved aircraft stability, reduced unwanted accelerations, and to some extent counteracted turbulence, making life in the astrodome a bit easier for the navigator.

Along with the advances in instrumentation, Thurlow publicized his work on the effect of Coriolis acceleration on the accuracy of bubble sextant readings. His work removed the last source of correctable error from aircraft sextant observations.[5]

Advances in Computation

Just as instruments for observation were evolving, so were publications and methods of computation. The United States

[5] For the sake of simplicity, several forms of sextant error, and the corrections for them, have been ignored throughout this review.

had experimented with an *Air Almanac* in 1933. The *Air Almanac* was a streamlined version of the *Nautical Almanac*, tailored for rapid access to the celestial data most useful to aviators. It was not published the following year. The rapid expansion of aviation in the years preceding the war caused a renewed demand for a specialized almanac for aviators. The Naval Observatory resumed publication of the *Air Almanac* in 1941.

Along with the new almanac came new tables of pre-computed solutions. H.O. 214, *Tables of Computed Altitude and Azimuth*, appeared in 1936. The tables were published in nine volumes, each covering a ten-degree band of latitudes. H.O. 214 saw extensive use during the war, serving mariners and aviators alike, and remained in use into the 1950s.

(See **Appendix C** for an example of solving the Navigational Triangle using H.O. 214.)

H.O. 218, *Altitude and Azimuth for Selected Stars*, appeared in 1943. These tables were created specifically for use on aircraft.

A few earlier methods of computation were revived for the war. One graphical method used in wartime aircraft was Weems' *Star Altitude Curves*. The star altitude curves were plotted on pages, three stars to the sheet. A navigator could determine a fix by selecting an appropriate sheet and observing the altitude of the three stars it charted.

Another graphical method used in wartime aircraft was the Astrograph. Similar in principle to Weems' *Star Altitude Curves*, the Astrograph was an optical instrument that projected the paths of celestial bodies from a spool of film down onto the navigator's charts.

Several wartime inventors offered intriguing new mechanical computing devices for celestial navigation. Frederick Hayes Hagner of Archbold-Hagner Instrument Laboratory, Inc., Drury A. McMillen, a Brazilian engineer, and Dr. W.F. Hiltner, working at the College of Engineering of the University of Washington in Seattle, all introduced sphere-based devices that served as miniature analogs of the Navigational Triangle.

One of Hagner's devices was essentially a hollow hemisphere mounted in the cockpit. A simple optical component directed the image of a celestial object onto the inner surface of the hemisphere, upon which graduated lines were engraved. The pilot peered into the hemispherical chamber and observed the progress of the image across the graduated surface, guiding the aircraft to keep the image on the proper track. Moveable indices provided references, and a clockwork mechanism compensated for the movement of the celestial reference in relation to the Earth.

McMillen's Sphero-Graphical Navigation System modeled the Navigational Triangle on the surface of a metallic dome. Movable arcs and circles, engraved with precisely graduated scales, surrounded the dome. By setting the arcs and circles to correspond to sextant observations of two different celestial bodies, one could obtain a fix on one's position on the surface of the miniature Earth. McMillen seems to have been as much a vocal promoter as he was an engineer. In January of 1943 an article in *Fortune Magazine* described McMillen's system. In the same month, *Time Magazine* lamented that the U.S. government refused to adopt McMillen's Sphero-Graphical Navigation System, which he claimed was superior to all existing methods.

Hiltner's prototype Navigator-Sphere was a 5-inch bowling ball nested within a set of graduated rings. Setting the rings according to the known values, one could create a model of the Navigational Triangle from which the unknown values could be measured. The principal fault with all of these instruments was scale. It was difficult to scale down the Navigational Triangle to a size suitable for use on an airplane while maintaining sufficiently accurate graduations.

The new publications, the revival of graphical methods, and the advances in radio navigation aids finally displaced the Bygrave slide rule. In a brief article in *The Navigator* in 1967 the author was enthusiastic in his description of Bygrave's slide rule, but could locate one only in a museum.

Slide rules for celestial navigation received sporadic attention after the war. Robert A. MacGregor and Edwin A. Beito described a navigation slide rule in the *Proceedings* of the U.S. Naval Institute in 1950. MacGregor had used the rule, made by Keuffel & Esser, on many wartime flights between the Aleutian Islands and the Kurile Islands. The rule appears to be a 10-inch Mannheim type with the following scales:

ST scale, on the top of the top rail.
S scale, on the bottom of the top rail.

S scale, at the top of the slide.
ST scale, near the top of the slide.
C scale, at the bottom of the slide.

D scale, on the top of the bottom rail.

For the celestial navigator, the slide rule's time had passed. Other slide rules, however, continued to serve aviators for decades to come. Examples include the ubiquitous Dalton Dead Reckoning Computer and its many variants (such as the E-6B described by Paul Sanik in the Fall 1997 *JOS*), and a broad array of specialized slide rules for computing corrected air speed, corrected altitude, time-and-distance, takeoff and landing, range-of-flight, center-of-gravity, and so on.

The Cold War Era

Post-war improvements of the turbojet engine enabled aircraft to operate at much greater speeds and higher altitudes. Greater speeds meant a need for increased aerodynamic efficiency, and the protuberant astrodome caused drag. More important, higher altitudes meant pressurized cabins that could experience explosive decompression if the astrodome failed, a significant risk for combat aircraft. The astrodome had to go.

The solution to the astrodome problem was the periscopic sextant. The Plath firm of Hamburg had produced a periscopic sextant in 1930, but it was not successful. A U.S. patent for a Panoramic Sextant was issued to Carl J. Crane and Thomas L. Thurlow in 1941, but it did not go into volume production. Victor Carbonara of the Kollsman Instrument Company developed the periscopic sextant that equipped U.S. military aircraft during the second half of the 20^{th} century. Everitt and others at Hughes developed a similar instrument in England. Even after other aids to navigation became commonplace in civil airliners, large military aircraft retained the periscopic sextant as a backup, in case other aids to navigation were not available.

Progress in aircraft technology and changes in the political environment brought forth fleets of intercontinental nuclear bombers during the Cold War. The new bombers carried complex Bombing and Navigational Systems (BNS) to guide them to the target. These offensive weapon systems included redundant subsystems to provide several independent sources of data for navigation. One of these subsystems was the astrotracker, an automated sextant developed by Carbonara and others at Kollsman.

In the 1960s the Strategic Air Command's Boeing B-52 "Stratofortress" heavy bombers could carry thermonuclear gravity bombs internally and North American Aviation's AGM-28 "Hound Dog" nuclear cruise missiles on under-wing pylons. Aircraft in that configuration carried three astrotrackers: one for the B-52 and one for each of the two missiles.

Publications reflected the new technology. The *Air Almanac* was expanded to include information about selecting stars for use with periscopic sextants and astrotrackers. In 1951 the U.S. Navy Hydrographic Office published H.O. 249, *Sight Reduction Tables for Air Navigation*. H.O. 249 was the latest refinement in tables for the aerial navigator.

The advances in electro-mechanical computing mechanisms that made possible the astrotracker were also applied to the solution of navigation problems. The Sperry Gyroscope Company and IBM, among others, developed complex electro-mechanical BNS computers that helped automate navigation. Periscopic sextants and printed tables were kept on board for backup if the computer failed.

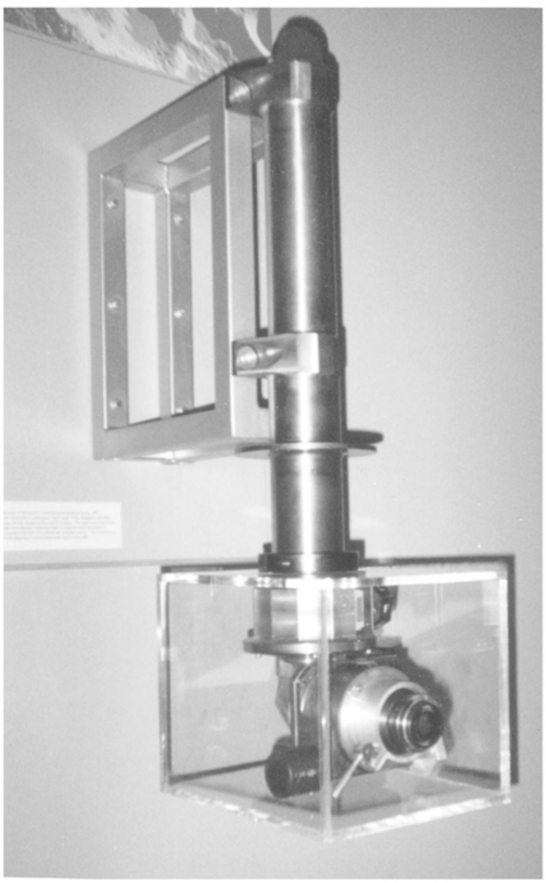

Figure 6. The space sextant in the MIT Museum.

Since the 1970s, advances in other fields have diminished our reliance on celestial navigation. Doppler radar, laser ring gyros, pocket-sized electronic digital calculators, and the Global Positioning System have replaced the slide rules and sextants on today's aircraft. The *Air Almanac* for the year 2007 is the last edition to be printed on paper. Beginning in 2008, the document will only be available on CD-ROM. Today, one can obtain the altitude and azimuth for the Sun, computed for any date, from the U.S. Naval Observatory's web site.

(See **Appendix D,** and compare the USNO results with those of the other methods.)

The sextant may still prove useful in space travel, however. Early U.S.-manned spacecraft were equipped with sextants. One such instrument was attached to a wall in the MIT Museum, just beside the door to the conference room where the Oughtred Society held its Fall Meeting on 12 November 2005.

In January, 2007, China successfully destroyed one of its defunct weather satellites with a ground-launched guided missile. As more nations gain the ability to incapacitate satellites, we may need to rethink our reliance on satellite-borne navigation systems. The sextant may live on. We will just have to wait and see what is in the stars.

Appendix A

Solving for Computed Altitude using the Cosine-Haversine Formula

for a Sun sighting at Westbrook, Connecticut, 1400 EST Saturday, 10 November, 2007.

Assumed Position 72 26 W, 41 17 N

1400 EST = 1900 GMT (UT)

From the Air Almanac 2007:

GHA Sun at 1900 = 109 01.3 Declination = 17 11.6 S

hav z = hav (L ∼ d) + cos L cos d hav t

[When declination is opposite latitude, as in this case, L∼d is interpreted as L+d]

calculate (L+d):

Latitude = 41 17

Declination = 17 12

(L∼d) = 58 29

calculate t (Meridian Angle):

GHA = 109 01

Longitude = - 72 26

t = 36 35

hav z = hav 58 29 + cos 41 17 cos 17 12 hav 36 35

lcos 41 17 = 9.8759 – 10

lcos 17 12 = 9.9802 – 10

lhav 36 35 = 5.9923 – 10

lhav x = 5.8484 – 10 , hav x = .0706 (lhav x and hav x are intermediate values

hav z = hav 58 29 + .0706 used only in the computation. Angle x

hav z = .2388 + .0706 need not be expressed as an angle.)

hav z = .3094

z = 67 35

Computed Altitude (Hc) = 90 – 67 35 = 23 25

(This method does not offer a simple solution for Azimuth.)

Appendix B

Solving the Navigational Triangle using the Bygrave Position Line Slide Rule

For a Sun sighting at Westbrook, Connecticut, 1400 EST Saturday, 10 November, 2007.

Assumed Position 72 26 W, 41 17 N 1400 EST = 1900 GMT (UT)

From the Air Almanac 2007:

GHA Sun at 1900 = 109 01.3 Declination = 17 11.6 S

calculate t (Meridian Angle):
GHA = 109 01
Longitude = - 72 26
t = 36 35
Set the Cursor to Zero on the Lower Scale 0 (Cursor against the mechanical stop)
Set the declination on the Upper Scale 17 12
Set t on the Lower Scale 36 35
Read the value of x on the Upper Scale 21 05
calculate b = 90 – Latitude
90 00
Latitude = – 41 17
b = 48 43
for contrary names, y = b – x
b = 48 43
x = – 21 05
y = 27 38
Set x on Lower Scale 21 05
Set t on Upper Scale 36 35
Set y on Lower Scale 27 38
Read Azimuth on the Upper Scale 38 02 (Converts to 218 02)
Set Azimuth on the Lower Scale 38 02
Set y on the Upper Scale 27 38
Set the Cursor to Zero on the Lower Scale 0 (Cursor against the mechanical stop)
Read h (Altitude) on the Upper Scale 22 24

Appendix C

Solving the Navigational Triangle using H. O. 214

for a Sun sighting at Westbrook, Connecticut, 1400 EST Saturday, 10 November, 2007.
Assumed Position 72 26 W, 41 17 N
1400 EST = 1900 GMT (UT)
From the Air Almanac 2007:
GHA Sun at 1900 = 109 01.3 Declination = 17 11.6 S
calculate t (Meridian Angle):
GHA = 109 01
Longitude = - 72 26
t = 36 35
From H. O. 214, Volume V:
Turn to the section for Latitude 41 , declination and Latitude opposite names: page 37
Select the Column for the declination nearest to 17 12 S
Enter the Row with the value of t nearest to 36 35
Note results:
Altitude _d _t Azimuth
23 05 88 47 142.3 (Converts to 217.7)
Use Multiplication Table inside rear cover to find Altitude Corrections for
declination – 00 10.2
t – 00 11.8
Difference in Latitude – 00 13.4
Total Altitude Correction – 00 35.4
Uncorrected Altitude = 23 05
Altitude Correction = – 00 35
Corrected Altitude = 22 30

Appendix D

Solving the Navigational Triangle using the U.S. Naval Observatory web site[6]

Astronomical Applications Dept.
U.S. Naval Observatory
Washington, DC 20392-5420
WESTBROOK, CONNECTICUT
o , o ,
W 72 26, N41 17
Altitude and Azimuth of the Sun
Nov 10, 2007
Eastern Standard Time
Altitude Azimuth
(E of N)
h m o o
05:30 -11.8 102.5
06:00 -6.4 107.2
06:30 -1.1 112.0
07:00 4.2 117.0
07:30 9.0 122.3
08:00 13.6 127.9
08:30 17.8 133.9
09:00 21.7 140.3
09:30 25.0 147.2
10:00 27.7 154.6
10:30 29.8 162.4
11:00 31.1 170.6
11:30 31.6 179.0
12:00 31.3 187.4
12:30 30.1 195.6
13:00 28.2 203.5
13:30 25.7 211.0
14:00 22.5 218.0
14:30 18.7 224.6
15:00 14.6 230.6
15:30 10.1 236.3
16:00 5.3 241.7
16:30 0.6 246.7
17:00 -5.2 251.6
17:30 -10.6 256.3

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This endnote is added to highlight the novelty of Leonard Charles Bygrave's bombsight invention, as described in G. B. Patent No. 124,857, *Sights for Use on Aircraft*, 10 April 1919. His remarkable design was suggested in 1918, while he was serving as a Temporary Lieutenant in the RAF. The device is unique in that its principal component is essentially a manometer. This may be one of the earliest applications of "fluidics". Briefly, the sight was based on the following principles: The bombing angle depends on both the altitude and speed of the aircraft. Two sighting points establish the correct line to the target. The sighting points are the tops of columns of fluid. The height of the fluid in one column is determined by atmospheric pressure that varies according to the aircraft's altitude. The height of the fluid in the other column is determined by pressure that varies according to the aircraft's speed. By using tubes that vary in diameter in the necessary proportions, a calibrated instrument could be made that automatically solved the bombing angle problem for all useful altitudes and speeds. Although proven to be impractical, the design was a delightful example of Bygrave's ingenuity and his creative approach to the problem.