
An Update on Log-Log Slide Rules Before 1910

Tom Wyman and Bob Otnes

Introduction

The purpose of this paper is to collect and republish earlier papers relevant to the early history of log-log slide rules.

First of all, recollect that generally the expression $x = a^b$ can only be evaluated using logarithms in the following manner:

$$\begin{aligned} x &= a^b \\ \log(x) &= \log(a^b) \\ &= b \times \log(a) \\ \log(\log(x)) &= \log(b) + \log(\log(a)) \\ x &= \log^{-1}(\log^{-1}(\log(b) + \log(\log(a)))) \end{aligned}$$

The authors remember doing this procedure in high school algebra using tables of logarithms. It was a pain. The chances of performing the process without error were small.

When the operation a^b is performed on a pocket calculator, rather than employing table lookup, it implements the above procedure employing approximations for the logarithm and inverse logarithm such as were defined by Hastings [4].

The authors would guess that most people pushing the a^b button on a calculator have not the foggiest idea what is going on in the device, and even if their lives depended on it, could not do the operation using the table lookup method.

In any event, the procedure is tedious to perform. Hence, log-log slide rules greatly simplified the operation and for a period of about 70 years, say 1900 to 1970, were widely employed to execute it, noting that log-log slide rules were not commonly available before 1900.

Chronology

- 1614** Napier publishes his book on logarithms of trigonometric functions.
- 1710** Ward [15] publishes his book on evolution and involution with applications to simple and compound interest. The methodology for log-log computation of powers and roots is well established by this time.
- 1814** Roget's paper on log-log slide rules [12] is read to the Royal Society and published in 1815.
- 1857** Rooker, J. & A. [1], present their implementation of Roget's linear rule to the Science Museum in London. The rule is shown in Figures 1 through 4 in [12] and reprinted in this issue.

1881 Thomson [14] patents his log-log slide rule. See [8] and below for a reconstruction of this rule.

ca 1900 Dunlop and Jackson [1] design their log-log slide rule for John Davis & Son. It is made by D&P [3].

1908 Yokota [16] patents his slide rule. It is made by Denner&Pape [3].

1909 Keuffel & Esser produce their first log-log slide rule, the 4092, with no author's name on the instruction booklet and no mention anywhere in their literature of the designer's name. Could the layout have been furnished to them by Dennert & Pape in Germany?

Peter Mark Roget, M.D.

The origin of slide rules specifically designed for use by engineers can be traced to James Watt (1736-1819), the Scottish engineer and inventor of the separate steam engine condenser. Surprisingly, a key contribution to the evolution of the engineer's slide rule was made not by an engineer but by the English physician Peter Mark Roget (1779-1869). He is best known for his *Thesaurus of English Words and Phrases* which went through twenty-eight editions in his lifetime. He was a remarkably talented man with diverse interests. In 1814, Roget's colleague, William Hyde Wollaston (1766 – 1828) M.D. and Secretary of the Royal Society, who had won the Society's prestigious Copley Medal in 1802 for his pioneering discoveries in chemistry, optics, and physiology, presented a paper by Roget to his peers.

In his paper, Roget introduced the concept of log-log scales for use in making exponential calculations involving fractional powers and roots. As a result of the enthusiastic reception to his paper, Roget was elected a Fellow of the Royal Society in March 1815, the same year that his paper was published in the Royal Society's *Philosophical Transactions*.

A biography of Roget, published earlier this year, *The Man Who Made Lists* by Joshua Kendall [5], is an entertaining read, but readers should be wary of misstatements such as the dust jacket blurb that asserts, "Peter Mark Roget lived a colorful life, full of unexpected twists and discoveries: he invented the slide rule." [!] On page 197 the author writes, "What Roget had invented was the log-log scale, the centerpiece of the modern slide rule, which remained in wide use until the rise of the pocket calculator in the 1970s." (Clearly, Kendall never used a slide rule or he would have recognized that log-log scales, instead of being "the centerpiece of the modern slide rule," were of little value to the majority of slide rule users.)

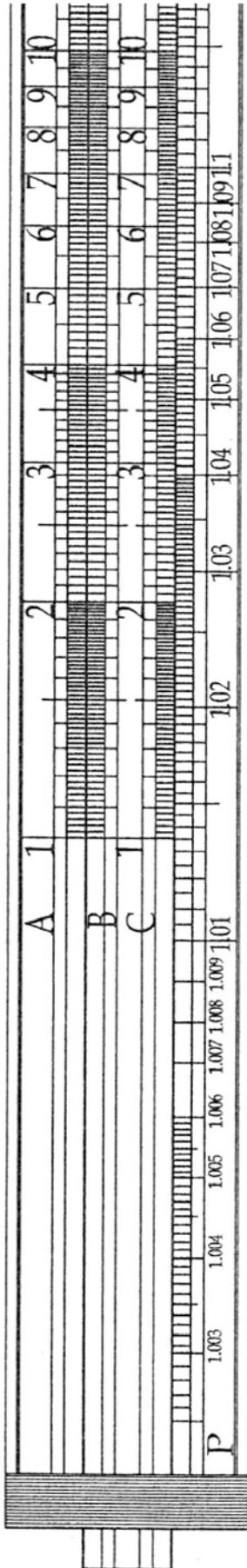


Figure 1a. Thomson Left.

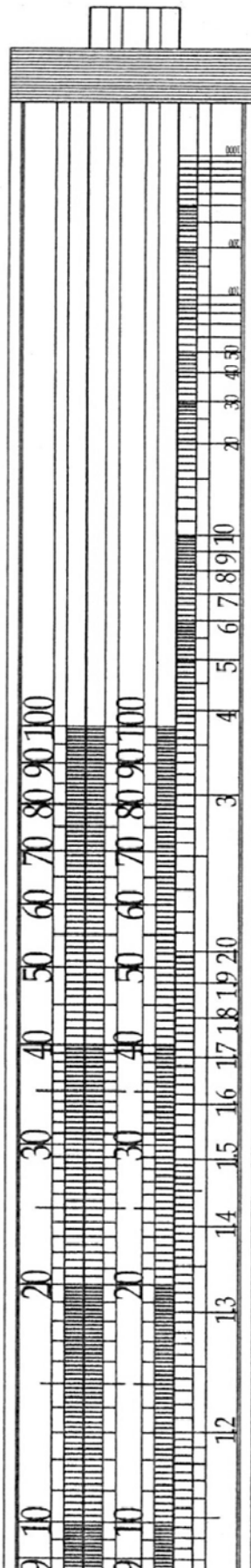


Figure 1b. Thomson Right.

Log-Log Rules Before 1910

Baxandall and Pugh[1] in their revised *Calculating Machines and Instruments* catalogue have a small section devoted to log-log slide rules. This has been the source of our information on log-log slide rule implementation prior to 1900 or so. Generally, the production of log-log rules in the 19th century was quite limited.

One possible candidate for inclusion that is not mentioned by the *Catalogue* in the log-log section as it is barely a log-log rule: the Bevan's engineering slide rule by Cary, ca 1817 [?]. It appears to evaluate the expression $y = (1.05)^x$, where 1.05 corresponds to an annual interest rate of 5%, x is the term of the loan in years and y is the result to multiply the principal in order to determine the amount of principal and interest at the end of the term. It is therefore more of a table than a full calculator. It does a similar calculation with the present value of an annuity. This article supposedly was written by de Morgan. The Interest Scale by Bate in the same *Catalogue* (page D33, item 105) would appear to be a more advanced form of this type of rule, noting that it was not examined for this article.

As mentioned in [8], the Science Museum (London) permitted the *Journal* to photograph the J.H. Thomson log-log slide rule of 1881 [1], page 46. The picture made by Dr. Bruce Babcock of the slide rule from these photographs is shown in Figures 1a and 1b. They were redone for this article by Dr. Babcock. Our thanks to him for this effort.

The Thomson rule has the first three scales (A, B, and C) in the form of standard two-cycle log scales, as found on the classical engineer's rule. What would be the D scale in such a rule has been replaced by a much longer log-log scale. This scale starts at about 1.003 and runs to about 100. This would work well for problems involving compound interest, perhaps less so for engineering problems.

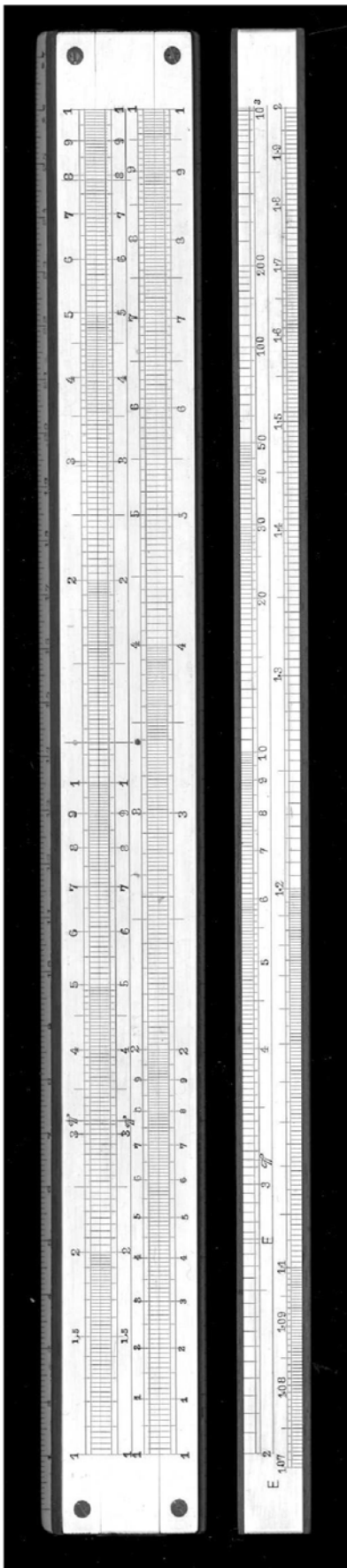


Figure 2. John Davis & Son, 1904.

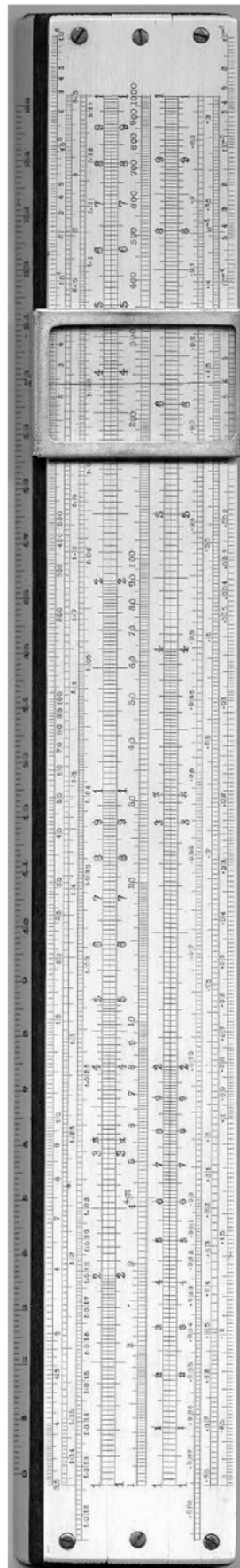


Figure 3. Yokota Rule.¹

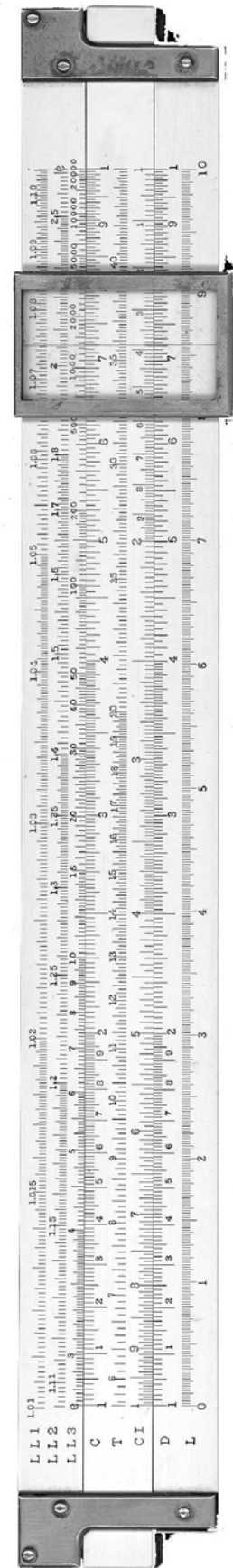


Figure 4. Keuffel & Esser, 1909.

¹as sold by John Davis & Son, 1908.

Moving on to the 20th century: After it surfaced in 1814, Roget's log-log concept was not widely implemented (see examples above) for ninety odd years until the period 1900 to 1909 when Dennert & Pape in Germany and Keuffel & Esser in America made the first commercially successful log-log slide rules for use in making comparatively sophisticated thermodynamic, electrical, and other engineering calculations involving transient phenomena and fractional roots. After the above log-log scales were successful, they appeared with increasing frequency on slide rules produced by other makers.

The rule sold by John Davis & Son is shown in Figure 2. It is item 160 in the *Catalogue* [1], where it is noted that it was designed by Lieut-Col H G Dunlop, RFA, and C S Jackson, MA. It is a basic Mannheim rule with an extra slide for log-log calculations. The construction of the rule and its slides differ a little from a more typical rule, in that the slides can be rotated about their axes.

With the special log-log slide in place it is limited to log-log calculations. The slide has a total of four log-log scales, two on each side. On the slide labeled "E", the combined two scales run from 1.07 to 1000 (more accurately, to 2^{10}). On the reverse side (labeled -E) their combined range is from .001 to .93+. Because it has nothing on the range 1.+ to 1.07, it probably would not be very useful for interest calculations.

Figure 3 shows the Yokota rule. In its English patent [16] of 1907 it states that "I, Seinen Yokota, Assistant Professor of Naval Architecture at the College of Engineering, Imperial University, Tokyo, Japan, ...". As will be seen, it is a Mannheim crowded on the front and bottom edge with many scales:

Top stator: LL3, LL2, LL1, and A

Slide: B, K, and C

Bottom stator: D, LL03, LL02, and LL01

Bottom edge: A 10-inch², decimally-divided scale, which, because the scales on the face of the rule are also 10 inches rather than 25 cm, can also be employed as an L scale relative to the D scale above it.

Back of slide: S, Sec and T

So, except for a CI scale this rule has all of the scales of a loaded Mannheim plus six log-log scales. Note that the main log-log scale starts at 3.5 rather than 2 or e .

The rule was first sold by John Davis & Son. However, we were told by Hans Dennert of D&P/Aristo [3] that it too was made by D&P and was sold by them until approximately WWII. A magnificent rule.

Figure 4 shows the Keuffel & Esser 4092 log-log duplex rule that K&E first sold about 1909. The main log-log

scale starts at e rather than 2 or 3.5 as is the case with the previous two rules. Even though it is a duplex, it has only 13 scales³. It does not have an L scale but does have a CI (inverted C).

It has two patent dates on it: June 5, '00 and Dec 22, '08. These two dates refer, respectively, to US patents number 651,142 and 907,373. The former is mainly for non-duplex rules; it does however, patent horizontal adjustment of their stators, while the latter appears to concern adjustment of the stators on a duplex rule. Neither patent mentions log-log scales.

There are a few mysteries with this rule. For example, there is no author's name on the original 4092 instruction booklet. Also, K&E apparently made no attempt to patent or copyright their design. And it is not known who designed it.

Dennert & Pape and Keuffel & Esser had a history of working together. Many early slide rules sold by K&E were made by D&P, and D&P made both of the John Davis & Son log-log rules. It is not impossible that D&P supplied K&E with their dividing engine technology if not the actual engines themselves. Again, it may be that K&E was also supplied with the log-log design. At this point in time there is no evidence either way⁴.

In any event, by 1909 the log-log slide rule was produced in volume in several forms. From this time on it would increasingly become part of the engineer's and scientist's tool box. While the slide rule has effectively been replaced by the many versions of the pocket electronic calculator, the so-called scientific versions of these invariably have a log-log function⁵.

Conclusion

Roget's concept was well in advance of its time. His seminal paper by someone remote from the field of mathematics merits wider circulation. For that reason, the *Journal of the Oughtred Society* has reproduced the paper in its entirety in this issue. This is a tribute to Dr. Peter Mark Roget and his remarkable contribution in the development of the slide rule, as well as in the field of lexicography. (As an aside, we venture to suggest that readers of this article have made more use of Roget's *Thesaurus* than they have of his log-log slide rule scale. However, we are prepared to stand corrected if evidence suggests otherwise.)

References

1. Baxandall, D. and J. Pugh, *Catalogue of the Collections in the Science Museum—Calculating Machines and Instruments*, London, Science Museum, (no date).

²Hans Dennert said that this was the only rule with 10-inch scales that D&P ever made.

³As compared with the Yokota, which has 12 and is not a duplex.

⁴There is, however, a great deal of unexplored K&E correspondence still in existence that might explain what happened.

⁵Also, the type of pocket calculator employed by real estate salesmen has a built-in, hidden log-log capability to perform the rather involved calculations required to compute the size of a month's payment based on the balance owed, the yearly interest rate, and the number of years in the loan. While the method of performing this calculation has been known for many years, the authors doubt that many people can execute it with pencil, paper, and a table of logarithms.

2. Cajori, F., *A History of the Logarithmic Slide Rule and Allied Instruments*, originally published in 1910 and reprinted in 1994 by the Astragal Press, Mendham, New Jersey. It also contains an update on the subject that Cajori wrote about sometime later.
3. Dennert, H., private communication to Otnes while in Köln, circa 1990. This occurred over a two-day period, Dennert coming to Köln especially for the purpose of discussing D&P history relative to slide rules. During this period he related that D&P had made slide rules for both John Dunlop & Sons and Keuffel & Esser. In particular, the ones for John Dunlop & Sons were of the log-log type.
4. Hastings, C., *Approximations for Digital Computers*, Princeton University Press, 1955.
5. Kendall, D., *The Man Who Made Lists—Love, Death, Madness and the Creation of “Roget’s Thesaurus”*, New York, G.P. Putnam’s Sons, 2008. Has only a few pages on log-log slide rules, and they seem to be lacking in cohesion and verity.
6. Napier, J., *The Construction of the Wonderful Canon of Logarithms, etc.*, translated from Latin by William Rae MacDonald, F.F.A., Edinburgh, William Blackwood and Sons, 1889. Reprinted by the Classics of Science Library, 1997.
7. Otnes, R.K., “Log-Log Scales,” *Journal of the Oughtred Society*, 1:1., February, 1992, p19.
8. Otnes, R.K., “The Thomson Log-Log Slide Rule”, *Journal of the Oughtred Society*, 5:1, March, 1996, p11.
9. Paulin, E., “Log-Log Scales”, *Dennert & Pape—Aristo—1872 to 1978*, edited by K. Kühn and K. Kleine, published by W. Zuckschwerdt, München, Verlag GmbH für Medizin and Naturwissenschaftler, 2004. The article is in German in the text and in English on an included CD. This important article describes a number of log-log schemes in detail.
10. Pickworth, C.N., *The Slide Rule: a Practical Manual*, New York (etc.), Van Nostrand, 1904. While many editions appeared both before and after this one, the 1904 edition appears to be the first that mentions log-log slide rules.
11. Pletts, “An Article from the Past on Log-Log Slide rules, Some Slide Rule Improvements,” *Journal of the Oughtred Society*, 8:2, March, 1999, p59. Reprinted from *The Proceedings of the Physical Society of London*, v. XXXIII, February 15, 1921.
12. Roget, P.M., “Description of a New Instrument for Performing Mechanically the Involution and Evolution of Numbers,” *Philosophical Transactions*, 1815, Part I, pp9-29. Note: the paper was read in 1814.
13. von Jezierski, D., “Dr. Roget’s Sliding Rule,” *Skid Stick*, No. 20, June 2006, p6.
14. Thomson, J.H., *Slide Rules &c.*, British Patent No. 5540, A.D. 1881.
15. Ward, J., *Clavis Usuræ; or, a Key to Interest, Both Simple and Compound*, London, printed by J.M. for William Taylor, 1710.
16. Yokota, S., *British Patent No. 18,218*, August 12, 1908.