Slide Rules with Hyperbolic Functions

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Introduction

Slide rules with hyperbolic function scales are of major interest to me because I used this type of slide rule in my first real job, as an engineer in the Vibration and Flutter Unit of Boeing in Seattle, beginning in 1946. My work was mostly involved in solving the flutter modes of the B-47, B-50, and B-52 bombers by matrix iteration. Other work was in the wind tunnel and in research involving vector solutions of the flutter matrix. I designed a six-inch-wide, and two-foot-long slide rule to sort out the various vibration modes coming from our strain gauge tests. My beginning job title was "Vibration and Flutter Computer". This was a perfect description, for a large part of my early tasks included endless complex number calculations. Later, I was promoted to the title of Engineer.

It was a fascinating job, and I often used my K&E 4083-3 Log Log Vector slide rule, which was always at my side. This was the Golden Era of slide rule use, for the main tools we engineers had were the slide rule and published tables of logarithms and functions. We used adding machines to add or subtract the logs for completing multiplication or division. Also, we had a few Marchant and Friden desk calculating machines that we could use for regular multiplication and division. However, these had no paper printouts, so every answer was copied by hand. In fact, everything we calculated had to be recorded by hand. What we know today as calculators and computers were many years away in the future.

Hyperbolic functions are encountered often by scientists, engineers, physicists, and mathematicians, and are used in a wide range of applications, from transmission lines to Einstein's theory of relativity. Growth blossomed in the early decades of the 1900s when industrial inventions employing formulas with hyperbolic functions were introduced. Then in 1921 the first patent for a slide rule with hyperbolic function scales was filed. Continued expansion of electronics pushed the need forward, and by the 1960s over 34 manufacturers had introduced over 109 different slide rules with these types of scales.

These numbers need explanation because other researchers will probably produce different figures. For example, the number of manufacturers was taken from the number of different names on the list of slide rules on the Oughtred Society website.[1] Some of these manufacturers produced the same slide rule under different names. If we eliminate these kinds of duplicates, we end up with somewhere around 25-28 different companies which produced rules with hyperbolic scales. This number may also be questionable, for there are numerous Chinese slide rules. It was not possible with the information available to determine if one or more companies in the Chinese market produced the same rule under different names.

The number given as 109 different slide rules with hyperbolic scales also needs defining. For each slide rule on the list, four major items are listed: These are: (1) name and number; (2) arrangement of the scales on the rule; (3) gauge marks on the scales, if any; and (4) gauge marks on the cursor, if any.

Not all slide rules have gauge marks on the scales and/or cursor. But these features are included in the listing because gauge marks add expanded calculating power to a slide rule. These four items are the only variables that separate the slide rules included in the listing. Slide rules with different names and scale placements are listed. Also, a slide rule with the same name and scale layout could be listed if, say, the gauge marks on the scales or the cursor were different. However, there is no attempt to include variants of differences in a logo design, name location, different cursor style, etc.

A Short History of Hyperbolic Functions

This short history has been adopted mainly from *Hyperbolic Functions (Smithsonian Mathematical Tables)* prepared by George F. Becker and C.E. Van Orstrand, 1909.[2] I reviewed a few other references on the history of mathematics to confirm the important dates and people involved.

Hyperbolic functions were not introduced until around the 1760s. However, it was some two hundred years earlier that the first and one of the most important applications of the functions now known as hyperbolic was made by Gerhard Kremer, 1512-1594, the Flemish geographer who was better known by his Latin name, Gerhardus Mercator. In 1569 he issued his Mercator's Projection map on which the loxodrome was a straight line. His projection resulted in the making of a map in which a straight line (the loxodrome) always made an equal angle with every meridian. This was a significant breakthrough in navigation. Its importance is evidenced by the fact that today all deep-sea navigation charts of the world have as their basis this projection. Mercator published his map without explanation, and it was left to others following him to discover the formulas he used. These were later found to be: $\lambda = gd(m/a)$, and $(m/a) = \ln \tan[(\pi/4) + (\lambda/2)]$, where λ is the latitude, m is the projection point in latitude λ, and a is the radius of the Earth. The term gd is called the *gudermannian* (after Chistoph Gudermann, 1798-1851). Of interest is the relationship of gd to hyperbolic functions. This was subsequently found to be

[1] **JOS Plus**
[2] Washington, DC, Smithsonian Institution, 3rd reprint 1924

as follows: $tan\ gd\ x = \sinh\ x$; $\sec gd\ x = \cosh x$; and $\sin gd\ x = \tanh x$.

The study of hyperbolic functions began when it was noticed that the area under the circle was the integral $\int \sqrt{a^2 - x^2})dx$, whereas the area under the hyperbola was the integral $\int \sqrt{a^2 + x^2})dx$. We note the two equations only differ by the sign. As the area under the circle could be obtained by using trigonometric functions such as $x = asin\theta$, it was decided that there should be some relation based on the area under the hyperbola. Also, the area under the hyperbola ($y = b/x$) was found to be related to the natural logarithm function. In fact the early name for natural logarithms was hyperbolic logarithms. This led some to think that there might be other relations, perhaps involving imaginary numbers and the trigonometric functions and the logarithm functions. These thoughts, of course, opened new windows and new paths to explore.

Coming back to the development of hyperbolic functions, we find they are derived from the formula for the area of what is called a hyperbolic sector. A picture of this sector is usually found in text books in graphical form using the formula for the hyperbola $x^2 - y^2 = a^2$. We probably do not need to go further with this explanation as almost every calculus book shows how hyperbolic functions are derived. For those of you interested in additional information we suggest you consult these other sources. It is sufficient to say here that the early ideas expressed above, as well as others, led to the understanding of hyperbolic functions as they came to be known. This knowledge emerged in the 1700s and 1800s through the contributions of the following pioneers.

Vincenzo Riccati (1707-1775) is noted as the actual inventor of hyperbolic trigonometry. In his publications in the 1760s he introduced the use of hyperbolic functions to obtain the roots of certain types of equations, particularly cubic equations. He adopted the notation $Sh.\phi_.$ and $Ch.\phi_.$ for the hyperbolic functions, and $Sc.\phi$ and $Cc.\phi$ for the circular ones.

Soon after Daviet de Foncenex showed how to interchange circular and hyperbolic functions by the use of $i = \sqrt{-1}$. (It was Euler (1748) who introduced the symbol i for $\sqrt{-1}$. Also, we cannot forget De Moivre's well-known contributions that were generalized by Euler to: $(\cos x + i \sin x)^n = \cos nx + i \sin nx)$. From Euler and these earlier efforts emerged the now familiar circular and hyperbolic equations: $\sin \alpha = -i \sinh i\alpha$, and $\cos \alpha = \cosh i\alpha$. These are matched by the converse equations: $\sinh \beta = -i \sin i\beta$ and $\cosh \beta = \cos i\beta$. Using the above equations, substitutions could be made readily between the circular and hyperbolic functions.

The first systematic development of hyperbolic functions was by Johann Heinrich Lambert (1728-1777). He adopted the notation we use today; sinh u, cosh u, etc., and introduced the transcendent angle (later renamed the gudermannian) using it in computation and in the construction of tables. He is credited with popularizing the new hyperbolic trigonometry that modern science finds so useful. It has been said that Lambert did for hyperbolic functions what Euler had done for circular functions. In 1830, Gudermann published an important paper followed by extended tables. Then Cayley, in 1862, in recognition of Gudermann's contributions, proposed the name gudermannian for the angle that Lambert called transcendent. The name, gudermannian, remains today.

Modern interest in hyperbolic functions was accelerated with the invention and commercialization of electricity. The widespread use of electricity started with the telegraph's development in the mid 1850s, and then was followed by the telephone, electric light, and the fast growing need of industry for power. Generating plants and transmission lines started to cross the American continent. The erection of transmission lines involves the application of hyperbolic functions.

A new academic degree called Electrical Engineer was the Universities' answer to the need of the business world to handle electricity on a large scale. Impetus in the use of hyperbolic functions was increased in 1884 when the A.I.E.E. (the American Institute of Electrical Engineers) was founded. Starting in the late 1800s more and more uses of hyperbolic functions could be found as the uses of electricity expanded. As the academic world took notice of this growth, it was aided by the publication in the early 1900s in both the U.S.A. and Europe of more extensive tables of hyperbolic functions. One of the important textbooks of the time was Prof. Kennelly's (Harvard).[3] Since 1900, applications of hyperbolic functions have blossomed into every scientific discipline.

Slide Rules with Hyperbolic Scales

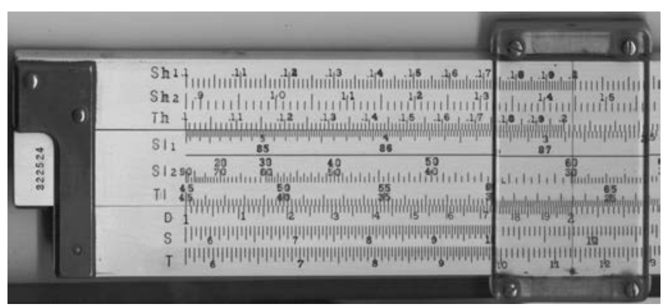

Figure 1. Back of the K&E 4093-3

As the expansion of applications accelerated, it seemed only a matter of time until the slide rule emerged as an aid for the electrical engineer. The history of slide rules with hyperbolic scales begins with a patent application on May 12, 1921 by Albert F. Puchstein. The title was *Device For Making Vector Calculations,* and included layouts of hyperbolic scales. In the patent application Puchstein says, "... my device is of such a nature that calculations can be readily made as to ... hyperbolic sines, cosines, tangents, etcetera, of vectors". It is interesting to note that

[3] Kennelly, Arthur E., *The application of hyperbolic functions to electrical engineering problems.* New York, McGraw-Hill, 1916.

a period of nine years went by from inception to introduction of the K&E 4093-3 in the market. For although the patent was approved three years later on March 25, 1924 (No. 1,487,805), the rule was not listed for sale in the K&E catalogs until 1930, when the manual was published. The rule's price was $16.00. The 4093-3S version, with a better leather case, was $16.85.

An historic slide rule, the K&E 4093-3 was the first slide rule with hyperbolic scales.

Next in line, it appears, was the Hemmi Model 153 (1933) that had gudermannian (spelled Gudermanian by Hemmi) scales for obtaining hyperbolic functions. Its 1934 manual describes this model as the Electrical Engineer's Universal Duplex Slide Rule With Patent Vector Scale, Gudermanian Scale and Log Log Scales. This is a very usable rule and is one of the most uniquely designed of all the hyperbolic slide rules.

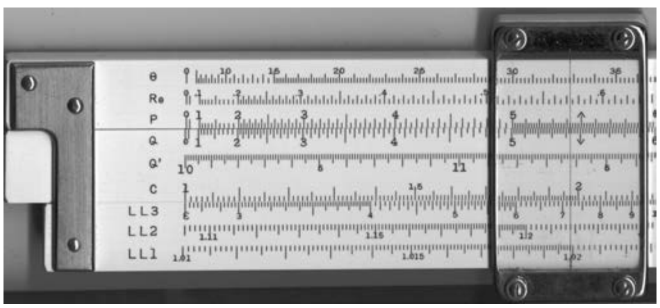

Figure 2. Back of the Hemmi 153.

The Hemmi 153 was the first hyperbolic slide rule with gudermannian scales design.

About another fifteen years passed before the Dietzgen Model 1735 and the Pickett Model 4 arrived on the scene. Their manuals are both dated with 1948 copyrights, although the rules were actually introduced in 1947. The European makers were a little slower in introducing slide rules with hyperbolic scales. The following dates are estimates of the earliest these manufacturers introduced their models:

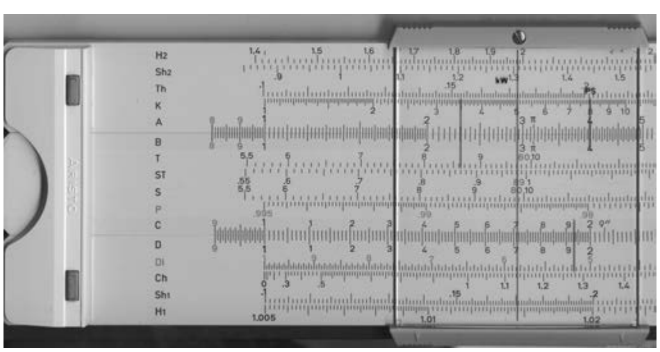

Figure 3. Back of the Aristo 0972.

- The Aristo HyperboLog 971 appears to be the first marketed in Europe in 1954.
- The Blundell JV56 Multi-Log Vector Duplex was second in 1957.
- About 1962 the Graphoplex 691a, Neperlog Hyperbolic appeared.
- The Faber-Castell 2/84 Mathema made its debut in about 1966.
- The date for the Reiss 3227, which was made in East Germany, also appears to be 1966.

These slide rules are known as high-end, as they usually are the most expensive and have the most scales of slide rules in a company's product line.

The Aristo HyperboLog 0971 had a successor that was the largest size hyperbolic slide rule produced and one of the most powerful. The Aristo HyperLog 0972 is shown here with its Ch, H and P scales.

Of interest is the fact that the first appearance (1954) of the hyperbolic slide rule in Europe was 24 years after its introduction by K&E in the USA. Nine years passed between 1930 and 1939; World War II was another 6 years; followed by more years during the rebuilding of Europe and Japan.

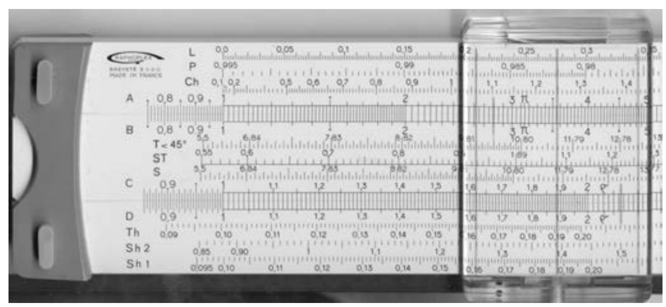

Figure 4. Front of the Graphoplex 691a.

The Graphoplex 691a Neperlog Hyperbolic is one of the most beautiful and has a Ch and P scale for added power.

During the later period, only limited shipments of slide rules occurred from the European and Japanese manufacturers to the U.S.A. As a result, K&E had a virtual monopoly on hyperbolic slide rule marketing in the United States for about 18 years. This continued from 1930 until around 1948 when Dietzgen and Pickett introduced their hyperbolic slide rules.

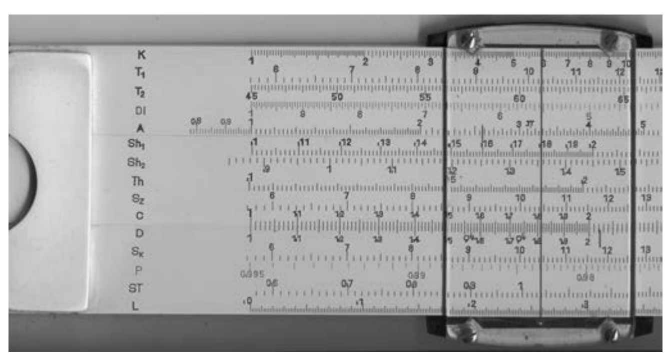

Figure 5. Back of the Reiss 3227.

The Reiss 3227 was made in East Germany, although "Made in Germany" is imprinted on it. The front side shows an unusual BI scale.

When we look at the worldwide sales of all types of slide rules, it does not appear that there was ever any real penetration by K&E or other U.S.A. slide rule makers into the European or Japanese markets. The same can be said about the European manufacturers regarding the U.S.A. market. Hemmi did have some limited success selling its own models in the U.S.A. before and after WW II. Also, some European cross production of U.S.A. slide rule names occurred at various times. But for all practical purposes, this was never on a significant scale. The U.S.A. manufacturer's names remained dominant throughout all of the years in the U.S.A. market, and the European manufacturers names remained dominant in Europe.

It might be mentioned, that to my knowledge, neither Dennert & Pape (1862) nor Nestler (1878) ever manufactured a slide rule with hyperbolic scales under their well-known names. Although D&P started using the product name Aristo in 1936, it was not until 1954 that the Aristo HyperboLog 971 was marketed. Also, the Faber-Castell Mathema 2/84 did not come on the scene until about 1966. I have no explanation as to why the European makers were so late introducing slide rules with hyperbolic scales. This is a similar puzzle as to why the U.S. Manufacturers never adopted the P scale into their designs. I thought the reasons might be due to patent infringement concerns. However, John Mosand (private communication) indicates that patent worries were not a problem. John says, "Clearly, there must have been conservative attitudes on both sides of the Atlantic." Maybe a reader can offer other reasons for these omissions, both for Europe and the U.S.A.

In the later years of slide rule production we find many names of hyperbolic Chinese slide rules on the list. In spite of what seems to be a large number of these that were manufactured, it appears few ever found their way to Europe or the U.S.A.

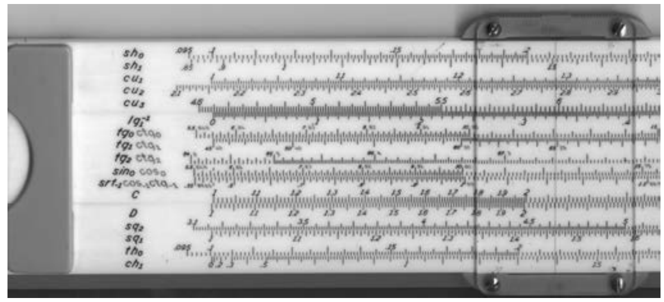

Figure 6. Front of the Flying Fish 1003.

Although not mentioned in the text, the Flying Fish 1003 is shown here, for it appears to have the most scales of all of the 25 cm. (10 inch) hyperbolic slide rules. It is a very powerful slide rule with 34 scales, including a Ch scale and three H scales.

The history of hyperbolic slide rules in the U.S.S.R. is surprising. A few years ago Andrew Davie referred me to a friend who was an expert in Russian slide rules. His name was Sergei Frolov, an engineer and computer expert, who has a website showing Russian slide rules. Sergei informed me that he had never seen a Russian slide rule with hyperbolic scales. This appears to be the case, as to date I have not seen one either.

Some Practical Pointers About Slide Rules With Hyperbolic Scales

The length of the scales on most of the rules is 25 cm or 10 inches. I found in actually measuring some of them that the manufacturers were not always correct in reporting the length of the scales. So I left the lengths as described rather than making changes. Both Hemmi and K&E produced larger 20-inch (50 cm) versions of some of their hyperbolic slide rules. I found only two pocket versions: the Flying Fish 1200 and the Pickett N4p in identical T and ES versions. One of the rarest is the Eckel Engineers Log Log Circular slide rule (size about 8 inches). It is the only circular slide rule known with hyperbolic scales. This was made by Eckel after he parted with Pickett.

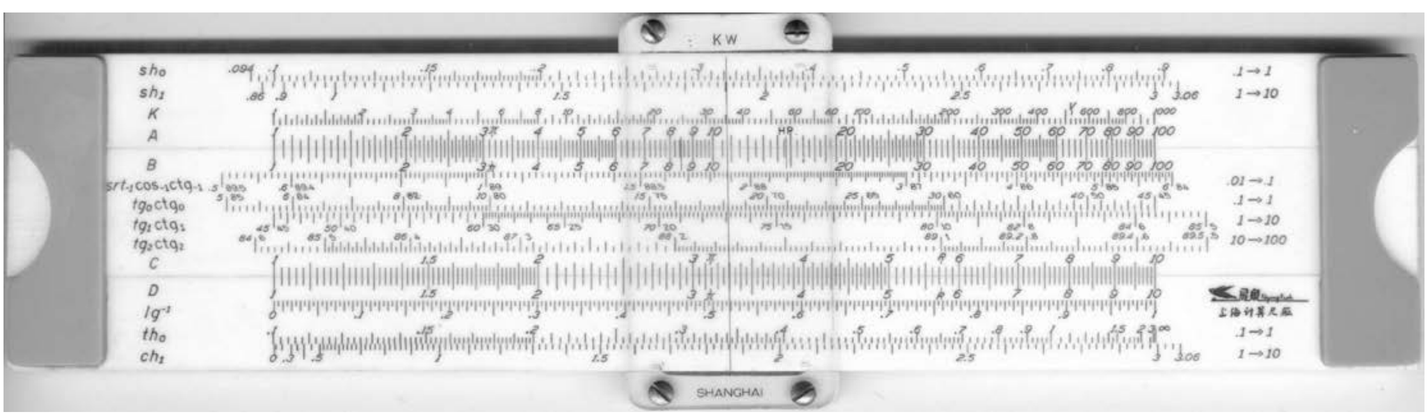

Figure 7. Face of the Flying Fish 1200.
A powerful pocket slide rule, the Flying Fish 1200 has both Ch and H scales.

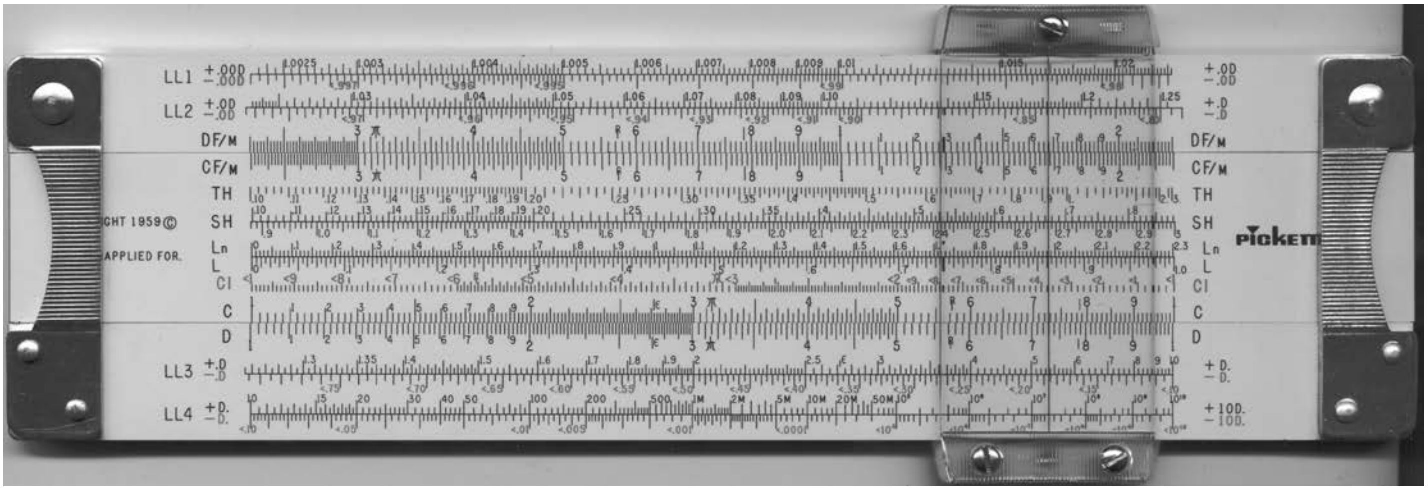

Figure 9. Back of the Pickett N4-p.

The Pickett N4-p is a good pocket slide rule with 34 scales, but sad to say, no Ch, H, or P scales.

Most of the rules have designated hyperbolic Sinh and Tanh scales. (A few exceptions are the Hemmi 153, and similar rules from other makers, that have gudermannian scales for obtaining hyperbolic functions). Usually the scales are shown as Sh1, Sh2, and Th. A few have a hyperbolic Cosh scale shown as Ch. Almost always there are two hyperbolic Sh scales to give a wider range for accuracy. The ranges of the hyperbolic scales usually are: Sh1 from 0.1 to 0.882; Sh2 0.882 to 3.0; and Th from 0.1 to 3.0. Only a few rules include a Cosh (Ch) scale in the layout. When the Cosh scale is included on the rule the range of values is usually Ch from 0.1 to 3.0. For those rules that do not have a Ch scale its value is calculated by use of the formula Ch = Sh/Th.

Next to consider is the layout of the scales on the various slide rules. Not all slide rules are created equal. Accuracy suffers if alignments of the scales and the cursor hairline become important considerations when calculations are made. It is obvious that accuracy could be lost if you have to turn the rule over to read the result, or rely on the hairline on the cursor to read from the top scale to the bottom scale. To minimize concern about alignment of the hairline with the scales, an ideal slide rule design would have the D, DI, Sh1, Sh2 and Th scales together on the bottom of the body, with these adjacent to C and CI scales on the slide. The CI and DI scales are required for calculating the reciprocal hyperbolic functions. The need for this layout is also obvious when Ch is calculated. The reason for this is that to obtain Ch you first have to find the Sh and Th values. Then in order to calculate Ch = Sh/Th you have to read the answer off the C scale. Of course this problem is solved if a Ch scale is included below the Th scale on the body. However, only five out of the one hundred nine rules on the list have a Ch scale. So most of the time the Ch value will have to be calculated using the Sh and Th values.

It was a surprise to find that in calculating Ch and the other reciprocal values, most hyperbolic slide rules were poorly designed. This is because the hyperbolic scales were not placed close to a set of C/D scales, and a CI or DI scale. Also, poor layouts were found where the Sh and Th scales are separated from top to bottom. There are many examples of poor design by the various manufacturers. One example of this is an early version of the K&E 4083-3 where there is no C scale on the same side as the hyperbolic scales. To read the Ch value, after getting the Th and Sh values, the manual instructions say to turn the rule over and read the answer at the cursor hairline on the C scale. K&E had poor instructions for this rule. However, for this particular K&E slide rule there is a much better alternative way to obtain the Ch value. All one needs to do, before working with hyperbolic functions, is to remove the slide, turn it over and reinsert it. This places the C scale on the opposite side next to the D scale. Also, you obtain the added advantage of the use of the CI scale for the reading of the Csch, Sech, and Coth values. The accuracy for this slide rule (and others with similar scale layouts) is greatly improved with this little trick.

With the slide reversed on the K&E 4083-3 the scale layout becomes almost ideal, and the accuracy and ability to handle all calculations involving hyperbolic functions is greatly enhanced. At almost a tie for ease of use is the Hemmi No. 153. This rule and three others on the list are gudermannian in design. With the slide reversed on this slide rule you can read off the Sinh and Tanh values on the T and Q scales with one setting on the Gθ scale. Then sliding the hairline to the Sinh value on the Q scale you can read the Cosh value on the Q' scale. Other rules that work well for hyperbolic functions with the slide reversed are as follows: Aristo 0971, Aristo 0972, Dietzgen 1735, Ding Feng 5471, Flying Fish 1004, all Pickett Model 4s, and SIDA Models 1, 1083 and 6201.

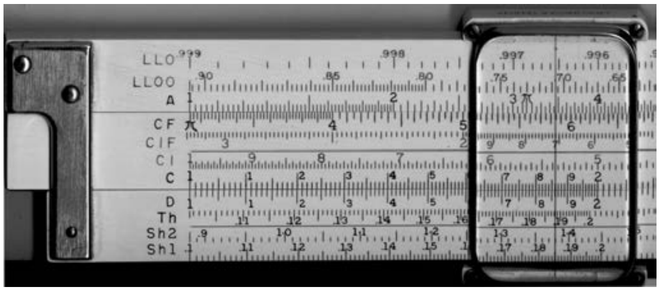

Figure 10. Back of the K&E 4083-3.

This is the back of the K&E 4083-3 with the slide reversed, bringing the C and other scales onto the same side as the hyperbolic scales.

My vote for the best designed actual layout (without the slide reversed) for a hyperbolic slide rule is the little known PATRICK Corp, MARK IV data log. It has the C/D, DI, Th, Sh2, and Sh1 scales all placed together on the bottom of the rule. Alignment problems are at an absolute minimum. The arrangement of the hyperbolic scales, together with the others on this slide rule, make it a design standout.

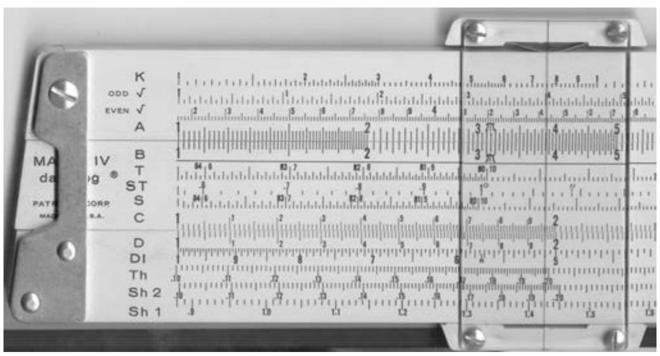

Figure 11. Front of the The Patrick Mark IV.

The Patrick Mark IV is a very interesting design that borrows features from the Pickett rules.

My vote for the best hyperbolic functions Instruction Manual is that for the Dietzgen No. 1725 (and No. 1735) rule. The instructions are almost identical for these two manuals and exceed any manual produced by other manufacturers. Another good one is the Dual Base Log Log manual for the Pickett Model 4. The Model 4 rule is an interesting design as the hyperbolic scales are on the slide and not on the body.

The usual procedure to find a hyperbolic function result is to start by moving the hairline to the value on the appropriate hyperbolic scale and then read the answer on the C or D or CI scale. We will use the K&E 4083-3 (with the slide turned over and reinserted so that we will have the use of an adjacent C scale) for showing examples. The following procedures may be used on almost all makes of rules. For our examples we will use the value $u = 0.4$ and show how to find all of the hyperbolic functions starting with $Sinh\ 0.4 = ?$. With this slide rule you move the hairline to 0.4 on the Sh1 scale and then read 0.411 under the hairline on the D scale. (Note that radians are used, not degrees, for entering the values on hyperbolic scales). At the same time, if the C and D scales are aligned, you can read $Csch\ 0.4 = 2.4$ on the CI scale under the hairline. Similarly, we find $Tanh\ 0.4$ to read 0.380 under the hairline on the D scale. At the same time you can read $Coth\ 0.4 = 2.63$ on the CI scale under the hairline. This slide rule does not have a *Cosh* scale. To find $Cosh\ 0.4$, move the hairline to 0.4 on the Th scale, slide 1 on the left side of the C scale to under the hairline, move the hairline to 0.4 on the Sh1 scale, and under the hairline read $Cosh\ 0.4 = 1.08$ on the C scale. At the same time you can read $Sech\ 0.4 = 0.925$ on the CI scale under the hairline.

Since the ranges of the scales are limited they cannot be used to find answers for all values of u. However, estimates may be used to find approximate results for low values ($u < 0.1$), and the slide rule used for high values ($u > 3.0$ of the hyperbolic functions. To do this the following estimates may be used:

When $u < 0.1$; $Sinh\ u = u$, $Tanh\ u = u$, and $Cosh\ u = 1.0$.

When $u > 3.0$; $Sinh\ u = (e^u)/2$, $Tanh\ u = 1.0$, and $Cosh\ u = Sinh\ u$.

Here is an example for $Sinh\ u$, where $u = 6.0$. We will use the C and LL3 scales with the slide in its usual position. On scale C, put the hairline on 6.0. Read 403.0 on the LL3 scale. Then divide by 2. So $Sinh\ 6.0 = 201.5$ (the actual value from the tables is $Sinh\ 6.0 = 201.7132$, and $Cosh\ 6.0 = 201.7156$). Then $Tanh\ 6.0 = (Sinh\ 6.0/Cosh\ 6.0) \approx 1.0$.

If you have a slide rule with hyperbolic scales and want to practice, you can the use the following table to check your calculations. It will give you an idea where the decimal points are to be placed when you are doing actual problems.

u	$Sinh\ u$	$Cosh\ u$	$Tanh\ u$	$Csch\ u$	$Sech\ u$	$Coth\ u$
0.2	0.2013	1.0201	0.1974	4.9668	0.9803	5.0665
0.4	0.4108	1.0811	0.3799	2.4346	0.9250	2.6319
0.8	0.8881	1.3374	0.6640	1.1260	0.7477	1.5059
1.4	1.9043	2.1509	0.8854	0.5251	0.4649	1.1295
2.4	5.4662	5.5569	0.9837	0.1829	0.1800	1.0166
2.8	8.1919	8.2527	0.9926	0.1221	0.1212	1.0074
3.0	10.0179	10.0677	0.9951	0.0998	0.0993	1.0050
6.0	201.713	201.716	1.0000	0.0050	0.0050	1.0000

The Slide Rule and Hyperbolic Functions of Complex Numbers

The hyperbolic functions of vectors in the form $(a \pm jb)$ are often found in problems. For example, the following formulas may be used to solve the two expressions $sinh(a \pm jb)$:

$\sinh(a + jb) = \sinh a \cdot \cos b + j \cosh a \cdot \sin b$, and
$\sinh(a - jb) = \sinh a \cdot \cos b - j \cosh a \cdot \sin b$.

where we are now using $\sqrt{-1} \equiv j$.

We find that the solution of a hyperbolic function of a complex number is a complex number. Often it is convenient to express hyperbolic functions of complex numbers in the vector form $A/\underline{\theta}$. The formulas for solving $\sinh(a + jb)$ in the vector notation $A/\underline{\theta}$, are $A = \sqrt{\sinh^2 a + \sin^2 b}$, and $\theta = \arctan(\cosh a \cdot \sin b/ \sinh a \cdot \cos b)$; or $\theta = \arctan(\tan b/ \tanh a)$. In the days before slide rules with hyperbolic scales, these formulas involved formidable calculations. One had to look up the hyperbolic functions in one set of tables and the circular functions in another. (Remember the hyperbolic functions are in radians, but the circular functions had to be converted to degrees before look-up). Often interpolations within the tables were required that added to the complexity of the calculations. (If one did not have a desk calculator (Monroe, Marchant, or Friden) to handle the multiplication and division operations in the formulas, one had to do the look-up in log tables). Log tables were an added calculating problem. Anyone who has worked with logs and antilogs knows that they often are confusing to use. Calculations are time-consuming, and one has to very careful not to make mistakes. The introduction of slide rules with hyperbolic scales presented a powerful tool for both making and checking these kinds of calculations, and in much less time.

Let us now show two ways to solve the formula; $\sinh(a + jb) = x + jy = A/\underline{\theta}$. The first example is by the hand-calculated method that had to be used before the introduction of the first hyperbolic slide rule in 1930. The second will be shown using the slide rule. Following are the steps of the first way:

Solve $\sinh(1.3+j0.57) = x+jy = A/\underline{\theta}$. ; where $a = 1.3$ and $b = 0.57$.

This may be solved using the following equations:

$x = \sinh\ a \cdot \cos b$, and
$y = \cosh\ a \cdot \sin\ b$, and
$A = \sqrt{(\sinh^2\ a + \sin^2\ b)}$, and
$\theta = \arctan(\tan\ b/ \tanh\ a)$.

As a and b are shown in radians we have to convert b to degrees for the sin b and tan b values. Converting b to degrees we have $(0.57 \times 180/\pi) = 32.659°$.

We now look up the values of the hyperbolic and circular functions in the tables.

So, we have:

$b = \sin 32.659° = 0.540$;
$\cos 32.659° = 0.842$;
$\tan 32.659° = 0.641$
And for $a : \sinh 1.3 = 1.698$;
$\cosh 1.3 = 1.971$; $\tanh 1.3 = 0.862$.

Substituting these amounts into the formulas above we find:

$x = 1.430$; $y = 1.064$; $A = 1.782$;
and $\theta = 0.640$ radians $\equiv 36.643°$.

Finally:

$\sinh(1.3 + j0.57) = 1.430 + j1.064 \equiv 1.782/\underline{36.643°}$

This example is not as simple as it looks, as in actual practice it is quite laborious to do. In order to complete this first calculation we had to look up the values of the hyperbolic and circular functions in different tables and interpolate between values where needed. (Most tables that were readily available at the time were limited in scope. So, the problem of interpolation was often encountered). We simplified the process by assuming there was a desk calculator available to do the multiplication and division. This eliminated the additional task to look up the logs of the hyperbolic and circular functions. Thank goodness for that, as the calculation using logs would be quite a job. Remember all of this work was done by hand using pencil and paper.

Compare the above with the following steps for finding the solution by slide rule: (We will use the K&E model 4083-3 slide rule without the slide reversed in working our example. The reason for not reversing the slide is that we are going to need to use both the circular and hyperbolic functions to complete the calculation. This K&E model is used because it is found in more numbers than the Dietzgen slide rules, or other company's models. However, we will use the instructions from the Dietzgen 1735 manual, as they are much better and easier to follow than K&E's manual):

Set 180° on CF to π right on DF
Opposite 0.57 on DF read 32.6° on CF.
Set 32.6° on T to 1.3 on Th.
Opposite right index of body read 36.6° on T (this is θ).

Now since θ (36.6°) is greater than $b = 32.6°$, the instructions tell us to do the following:

Set red 36.6° on S to 1.3 on Sh2.
Then opposite red 32.6° on S read 1.783 (this is A).
So, $\sinh(1.3 + j0.57) = A/\underline{\theta} = 1.783/36.6°$.

We can solve $\sinh(1.3 + j0.57) = x + jy$ for the values of x and y by using

$A/\underline{\theta} = 1.783/36.6°$

and the following steps: set the right index of slide to 1.783 on D. Then opposite red 36.6° on S read 1.43 on D (this is x). Set the right index of slide to 1.783 on D. Then opposite black 36.6° on S read 1.06 on D (this is y). So

$\sinh(1.3 + j0.57) = x + jy = 1.43 + j1.06;$

and

$A/\underline{\theta} = 1.783/36.6°.$

It is obvious that the steps using a slide rule with hyperbolic scales are much easier than trying to calculate by using interpolated values from published tables. Of real importance is the fact that it does not take very many sample calculations with the slide rule to find that one masters these steps quite readily. One can imagine how happy those working with hyperbolic complex functions were to see a slide rule that could free them from the table look-up routine.

There was another advantage of real importance. The table look-up routines were used when more accuracy was needed than could be obtained from the slide rule. However the hyperbolic slide rule could be used to quickly check the answers obtained from the long calculations made by using the tables. As time passed, these slide rules became the leading tool for both making and checking calculations involving both hyperbolic and hyperbolic complex functions.

The List of Slide Rules with Hyperbolic Scales

This list, which is a table of data for 109 slide rules, is presented as a **JOS Plus** feature on the website of the Oughtred Society. The Journal will be using **JOS Plus** in coordination with its articles to present information that would be difficult to read in print and to present additional information when needed. Please visit our website www.oughtred.org to view this complete list of hyperbolic slide rules.

This list is the first draft of what we hope will become a fairly complete record of slide rules with hyperbolic scales. To complete it, I am asking for help from the International Slide Rule Group (ISRG) members, the Oughtred Society (OS) members, and other slide rule lovers to fill in the missing pieces. First, please look at the list for errors, and let me know what changes should be made. Second, let me know the name and details for any slide rule that has been omitted and should be added. I will be most happy to acknowledge the contributions that anyone makes to improve the listing. My contact information appears at the end of the list on the website.

The final goal is to include every known slide rule with hyperbolic scales for each manufacturer. In listing the rules that have the same name and number, I'm only looking for variations of the placements of the hyperbolic scales or other scales on the rule. Also, I'm trying to include variations in the gauge marks on the scales and cursor.

The list is in alphabetical order by manufacturer. I have included only actual details that have been verified. Front Scales denote the side of the slide rule that shows its name or number. This is not necessarily the side containing the hyperbolic scales. Red color denotes scales that are in red color on the slide rule. Where red is missing in some of the listings, it is because I do not own the rule or do not have a picture that shows the color of the scales. For some rules I do not have pictures that show all of the gauge marks on the scales and cursor. At a later time, I am hoping to add front and back pictures for all of the rules to this archive.

I wish to acknowledge the contributions made by Michael OLeary, Pierre Vander Meulen, and John Fahey. Michael sent me emails with large-size pictures of the variations of both K&E and Pickett rules that he had. Pierre did the same for many different Chinese slide rules that he had purchased when he worked in China. John mailed me pictures of the Eckel circular slide rule. These, and other sources, were all important to recording an accurate listing for these rules. At the end of the listing (on the website) all of the sources of information are shown, in order to give each of them proper credit. In the listing these sources are identified by initials at the far right for each rule.