Gunter Rules in Navigation[1]

Otto van Poelje

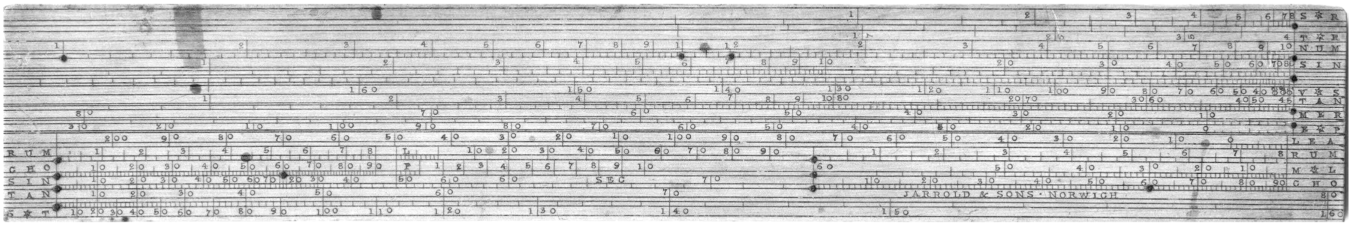

Figure 1: One foot Gunter Rule

Introduction

The Gunter rule is a legendary and much sought-after object for collectors of slide rules. Legendary, because of its naming after the inventor of the logarithmic scale (Edmund Gunter) and for its usage with chart dividers or compasses, but also mysterious because of its many exotic scales. Some slide rule collectors already have given their attention to the Gunter rule in references [22] to [25].

This article will further address the Gunter rule, with its background, its scales, and its application in navigation at sea. Also some other navigational rules from that period will be described.

Navigation in Gunter's Time

Early 17th century was a turning point for navigation, from "plain sailing" on sea charts with rectangular grids, to the use of Mercator charts for plotting compass courses as straight lines (the so-called "loxodromes"), see [20]. In mathematics, the field of trigonometry and spherical goniometry was already well known. By astronomical observations of the altitude of sun or Polar star with the cross-staff (the precursor of octant and sextant), the latitude of a ship's position could be found. The longitude was a much more difficult problem, only to be solved later by accurate chronometers, better calculation methods ("Line of Position"), and eventually by wireless technology like Decca, LORAN and GPS. The challenge for the sailors of that time was to introduce the new techniques in their navigation methods. The practice of solving navigational problems by construction of course lines on the sea charts was to be supplemented by the newly invented calculation methods using logarithms.

Definition

When is a rule a Gunter rule? More than one answer is possible, but the main criteria appear to be that it is a fixed rule, without moving parts, and that it has one or more logarithmic scales. The inherent assumption being that with these characteristics it is possible to multiply or divide by moving scale distances between the tips of the dividers. This is the most generic definition for a Gunter rule.

Now there exists one specific type of Gunter rule which collectors encounter very often in their search for new acquisitions. In my own contacts with fellow collectors about Gunter rules, I know of some hundred specimens of that type. Also they can be found in many museum displays on slide rules. This specific type could be described in pages full of definition text, but the easiest definition is by graphical example: the best picture of the most often encountered Gunter rule can be found as a foldout drawing in [22], drawn by Bruce Babcock. This we will call the "Standard Gunter Rule". It has an amazingly large number of scales: 22 in total, including a "diagonal scale" for determining exact line lengths by means of dividers. The Standard Gunter Rule is most often constructed of wood, but sometimes of brass or German silver, and 1-foot specimens of ivory have been sighted.

The majority of known Gunter rules do not bear a maker's name or date.

Of course variations are encountered, like scales with different name abbreviations, extended scales (like the "Donn"-variant with square, cube, and military scales, see [14]), but also different sizes. Most Gunters are 2 feet by 2 or 1.5 inches. One-foot models exist, with the same Standard Gunter Rule scales compressed in this smaller size. Still the Standard Gunter Rule definition covers an amazingly large proportion of all known Gunter rules.

History of Calculation by Logarithms

Throughout history there has been an ever-increasing need for calculations. For example, at the end of the 16th century, Kepler had published his laws on planetary motion, and this knowledge resulted in the need for massive calculations to determine the orbits of our planets. But the art of multiplication and division was tedious handwork, only known to professional "calculators". An approach existed, even then, to transform the operands x and y of a multiplication, into a goniometric domain with the formula

$$x \cdot y = \sin(a) \cdot \sin(b) = 1/2(\cos(a-b)\cos(a+b)$$

where $a = \arcsin(x)$ and $y = \arcsin(y)$. This allowed the

[1]Revised and extended from a paper in *The Proceedings of the 9th International Meeting of Slide Rule Collectors,* The IM2003, held September 19–20th in Breukelen/Amsterdam, The Netherlands.

multiplication to be replaced by plus and minus operations in combination with sine and cosine tables (which were available at that time) for the necessary transformations.

The search for logarithms had the objective of finding a simpler transform:

$$\log(x \cdot y) = \log(x) + \log(y)$$

Three men of science played a major role in the history of the Gunter rule:

- John Napier (1550-1617), the first to publish the concept of logarithms and their corresponding tables in 1614
- Henri Briggs (1556-1630), calculator and author of the first decimal-based logarithmic table in 1617
- Edmund Gunter (1581-1626), inventor of the logarithmic scales (published in 1624)

These men had all the characteristics of scientists of that time: they could read, write, and probably converse, in Latin. They had a very wide and universal range of interests (unlike today's specialized scientist). For example, Gunter was active in theology, surveying, astronomy, navigation, sundial design, and even the time-dependency of the earth's magnetic field variation. He was more decimal-oriented than today's average Englishman, because he divided his own surveying unit, the "Gunter Chain" of 22 yards, into 100 "Links"; he also proposed to divide each of the 360 degrees of a circle into decimal fractions, instead of sexagesimal minutes and seconds.

At that time, speed in publishing was not our "internet speed": it could take many, many years before study resulted in publication, which even then was limited to a Latin-reading public.

John Napier worked for almost 20 years on his concept and tables of logarithms before publishing the results in his famous *Mirifici Logarithmorum Canonis Descriptio* (Description of the Miraculous List of Logarithms), see [1]. His logarithm, here called LN, would be written in today's notation as

$$LN(x) = -10^7 \ln(\sin(x))$$

The factor of "ten million" was related to the fact that integer numbers were favored in tables (because decimal fractions, though already known in principle, were not common knowledge, and even had various different and conflicting notations).

The natural logarithm ("ln") in this formula turns out to be the consequence of Napier's numerical approach for calculating logarithms. He described his logarithms in a kinematical model, and not in terms of the power of a number. Only a century later the mathematical context would be created for the "ln (x)" as an analytical function, with Euler's constant e = 2.718... as base number.

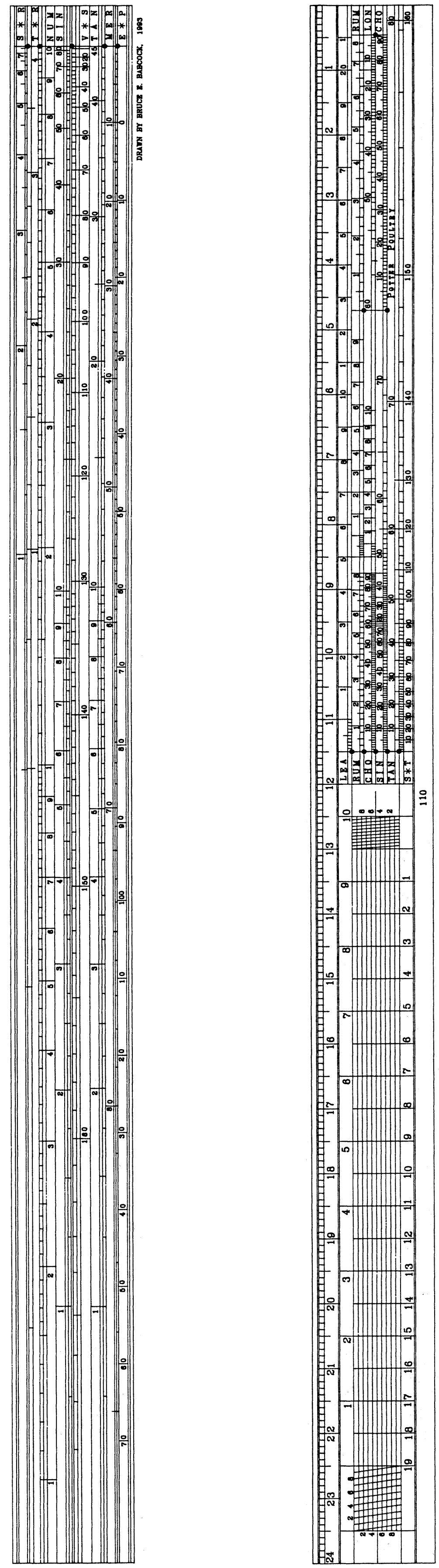

Figure 2: Standard Gunter Rule.

It is remarkable that Napier's tables only addressed the logarithms of sines, and not the logarithms of numbers. Napier assumedly gave priority to goniometric calculations as needed in astronomy and navigation, before his poor health prevented him from calculating other tables.

Henri Briggs, a Professor of Geometry at Gresham College in London for many years, was deeply impressed by Napier's publication in 1614, and he traveled to Edinburgh twice for lengthy discussions with Napier on the continuation of his research. They agreed to get rid of the factor of "ten million" (resulting in tables containing decimal fractions with Napier's "dot" notation), and to use the base of 10 for future log tables, which Briggs took it upon himself to calculate.

Soon after Napier's death in 1617, Briggs published his first decimal log table [2], from 1 to 1000 (the first "Chiliad", in his classical terminology). Later the tables were extended by himself and others. Still, Briggs only published logarithms of numbers, and not of goniometric functions like Napier had done. This means that a formula like

$$\frac{X}{\sin(x)} = \frac{Y}{\sin(y)}$$

could not have been calculated directly using Napier's tables, nor by Briggs' tables. And that calculation happens to be one of the most important formulae of plain navigation at sea, the Rule of Sines.

Enter Edmund Gunter

When Gresham College needed a new Professor of Astronomy in 1619, Briggs recommended his friend Gunter for the vacancy. The two of them must have had close interaction on the subject of logarithms. Unlike an ivory-towered university, Gresham College held public lectures and disputes in the English language, and Gunter must have received a lot of practical feedback from ship captains and other navigators in his audience.

Many of his known accomplishments are related to navigation at sea: for example, he invented the log line to measure a ship's speed (with "knots" for direct reading of the speed measured), and Gunter's quadrant, an astronomical altitude measurement and calculation device, related to the older astrolabium.

Gunter must have realized that navigational calculations would benefit from logarithms of both numbers and goniometric functions. In 1619 he published the first combined table [3], the "Canon Triangulorum", containing his own newly calculated decimal logarithms of sines and tangents, but he also added Briggs' logarithms of numbers. Now, at last, the Rule of Sines could be calculated by logarithms from tables in a single book.

Still, the goniometric tables in Gunter's book look strange to us, because he had added the term 10 to every entry in order to prevent negative numbers, which were not fashionable in his time. For example, $log(sin(30)) = -0.30103$, but in Gunter's table it was converted to 9.69897.

For general use, this would cause an incompatibility with Briggs' tables, but in Gunter's calculation examples the term 10 was always cancelled out by the fact that only ratios of sines were involved. This practice has survived in logarithm tables, even into the 20th century.

Gunter's Line and Gunter's Scales

Gunter took the matter one step further. He must have felt that some navigational problems needed an easier and faster solution than calculation by tables could provide, for example in coastal water navigation, in "haven-finding" situations.

As can be seen in Gunter's book [4] on various navigational instruments (1624), he was well aware of the "cross-staff" and the "sector" (in other languages called "proportional" instruments), and of the efficient use of their many already-existing scales. The use of dividers for making calculations on a sector was a practice already known in the 16th century (the invention of the sector is sometimes attributed to Galileo, but not conclusively).

M	*Sin.* 30.		*Tang.* 30.		
30	9.7054689	9.9353204	9.7701485	10.2298515	30
31	9.7056833	9.9352459	9.7704373	10.2295627	29
32	9.7058975	9.9351715	9.7707261	10.2292739	28
33	9.7061116	9.9350969	9.7710147	10.2289853	27
34	9.7063256	9.9350223	9.7713033	10.2286967	26
35	9.7065394	9.9349477	9.7715917	10.2284082	25
36	9.7067531	9.9348730	9.7718801	10.2281199	24
37	9.7069667	9.9347983	9.7721684	10.2278316	23
38	9.7071801	9.9347235	9.7724566	10.2275434	22
39	9.7093933	9.9346486	9.7727447	10.2272553	21
40	9.7076064	9.9345738	9.7730327	10.2269673	20
41	9.7078194	9.9344988	9.7733206	10.2266794	19
42	9.7080323	9.9344238	9.7736084	10.2263916	18
43	9.7082450	9.9343488	9.7738961	10.2261039	17
44	9.7084575	9.9342737	9.7741838	10.2258162	16
45	9.7086699	9.9341986	9.7744713	10.2255287	15
46	9.7088822	9.9341234	9.7747588	10.2252412	14
47	9.7090943	9.9340482	9.7750462	10.2249538	13
48	9.7093063	9.9339729	9.7753334	10.2246666	12
49	9.7095182	9.9338976	9.7756206	10.2243794	11
50	9.7097299	9.9338222	9.7759077	10.2240923	10
51	9.7099415	9.9337467	9.7761947	10.2238053	9
52	9.7101529	9.9336713	9.7764816	10.2235184	8
53	9.7103642	9.9335957	9.7767685	10.2232315	7
54	9.7105753	9.9335201	9.7770552	10.2229448	6
55	9.7107863	9.9334445	9.7773418	10.2226582	5
56	9.7109972	9.9333688	9.7776284	10.2223716	4
57	9.7112080	9.9332931	9.7779149	10.2220851	3
58	9.7114186	9.9332173	9.7782012	10.2217988	2
59	9.7116290	9.9331415	9.7784875	10.2215125	1
60	9.7118393	9.9330656	9.7787737	10.2212263	0
		Sin. 59.		*Tang.* 59.	M

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Figure 3: Page from *Canon Triangulorum.*

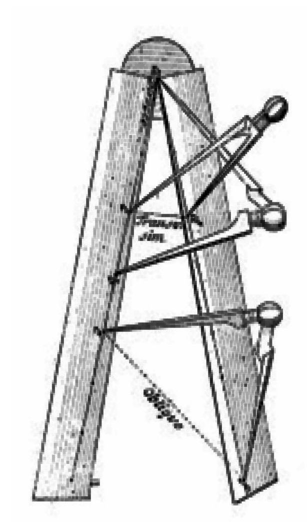


Fig. 4: Sector with dividers.

From there, it must have been a logical step for him to design a new type of scale, where numbers were represented by logarithmic scale distances, and where dividers were used to add or subtract those distances in the logarithmic domain. He called his scale the "Line of Numbers", other people used the term "Line of Proportion" or "Gunter's Line". His scale should not be confused with the "Line of Lines", the fundamental linear scale on a sector.

Actually, Gunter proposed in [4] three types of logarithmic scales, not only the Line of Numbers, but also the logarithmic sine and tangent scales, so that the Rule of Sines could be calculated between these scales; he also hinted that the addition of a "versed sine" scale might be easier for calculating the sides of a spherical triangle.

Gunter called any logarithmic scale "artificial", because that was the term Napier originally used, before he replaced it with the term "logarithms".

The power of Gunter's concept is proven by the fact that his two-cycle Line of Numbers and the corresponding SIN scale have maintained exactly the same structure as the A/B-scale and the S-scale of industrially-produced slide rules until the early 20th century.

These Gunter scales were proposed and described in the chapter "on the Cross-staff", as it appeared that this navigational instrument for measuring altitude of sun or stars still had space available for additional scales. Gunter did not mention his artificial scales in the chapter "on the Sector", and we don't know the reason for that: maybe he found one description was enough, the one in the Cross-staff chapter, or might he have had other reasons? However, none of the known cross-staffs reported in [21] have Gunter's lines or scales engraved at all.

On the other hand, an "English" variant of the sector is known, also called Gunter's Sector (although it is not mentioned in the original chapter "on the Sector" in Gunter's book), on which Gunter's three basic scales (marked as T, S, N) are available at the bottom of one side, when the instrument is opened.

This version was produced until the end of the 19th century, and can quite often be found at fairs or auctions, mostly in ivory.

Gunter's publications were so popular that they were reprinted many times, even after he died in 1626. In the 1673 reprint [10] many appendices had been added, for example one after the chapter "on the Sector", by Samuel Foster: "The Sector Altered; and other Scales added: with the Description and Use thereof". In this additional chapter (page 161) the addition of the three Gunter lines is mentioned: *"The Sector then being opened and so made a streight Rular; the outer edge hath inscribed upon it the three usual Scales of Logarithmetical Numbers, Sines and Tangents."*

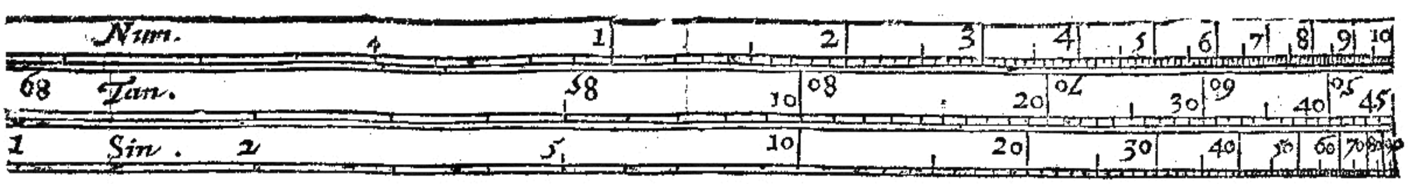


Figure 5. Original drawing of Gunter's Scales.

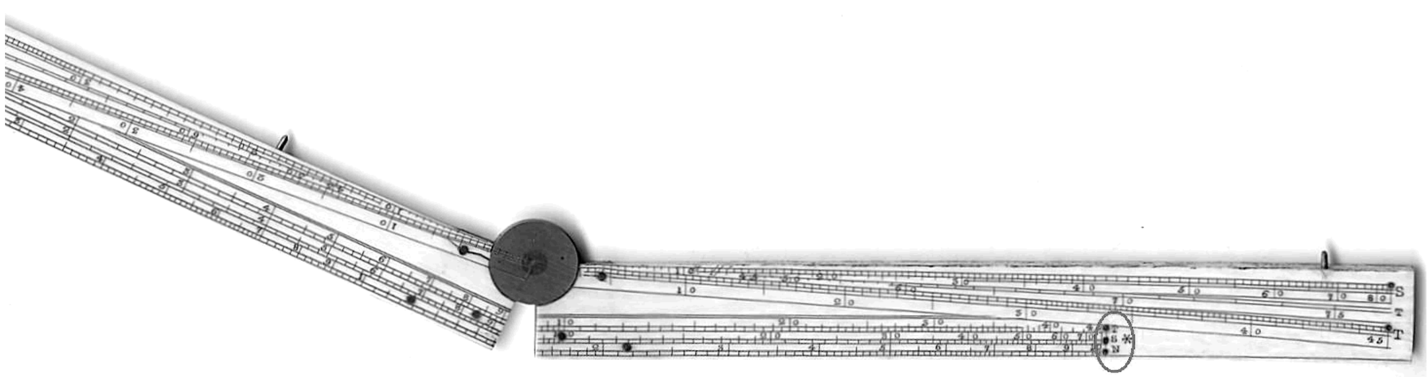
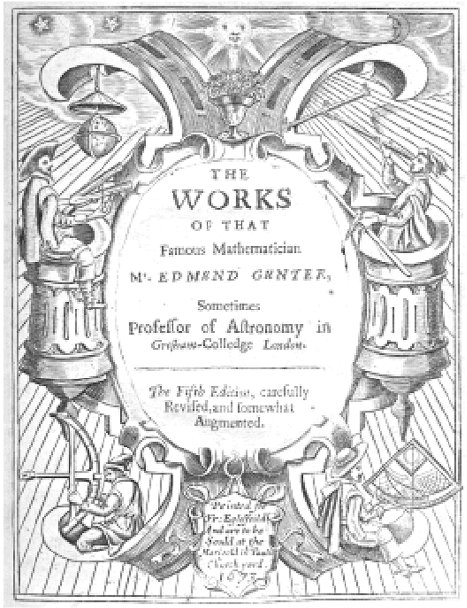


Figure 6. Gunter's scales on the English Sector.



THE WORKS OF THAT Famous Mathematician Mr. EDMUND GUNTER, Sometimes Profeſſor of Aſtronomy in Greſham-Colledge London.

The Fifth Edition, carefully Reviſed, and ſomewhat Augmented.

Figure 7. The 1673 edition of Gunter's works.

This looks very much like the English sectors that collectors are still finding today.

In this supplemented reprint, the Standard Gunter Rule could have been mentioned and described, if it had existed at that time, but it is not there.

The fact that usage of Gunter's basic three scales on a straight and plain rule was not mentioned in the 1673 edition, might suggest that the Standard Gunter Rule was introduced only later at the end of the 17th century or even 18th century. We must consider that changes happened slower in those days, and especially among sailors who were reputedly slow—see also [16]—in the acceptance of "high-tech" innovations, as the Gunter scales must have seemed to them.

It is very well possible that someone other than Gunter had realized the possibilities of these new scales on the surface of a plain 2-foot rule. The advantage would be that there is ample space for many scales on the two sides of a 2-inch-wide rule, with the increased accuracy of a 2-foot scale length. The absence of moving parts would be an additional advantage in a ship's rough environment, and a long rule is useful anyway on a chart table. This would mean that the Gunter Rule is named after Gunter only because his Line of Numbers and his artificial SIN and TAN scales are included, not because he invented the rule "in toto". In that case, the still remaining question is, who then did design the Gunter Rule, using the scales in Gunter's book, and when? Any contribution toward answering this question would be greatly appreciated.

The Scales on a Gunter rule

When we look at the Standard Gunter Rule, we see two faces containing the following information:

Front: a "diagonal table" on the left side in 19 half-inches, and a number of "plain" (mainly goniometrical) scales on the right side, none of which are logarithmic.

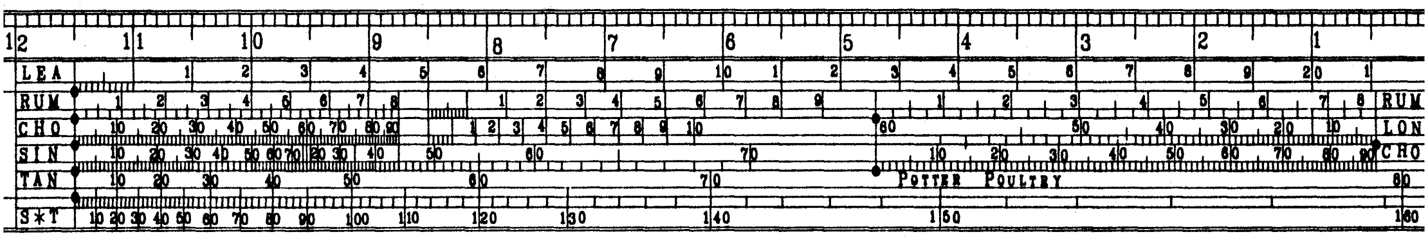
Back: a set of full-length scales, most of which are of a logarithmic ("Artificial") nature.

Many of these scales have been described extensively in Bion [11] with respect to construction and usage. The next table gives a summary of name and meaning of the main scales (the "formula" column gives the proportional length on a scale for a number marked X on that scale).

Front (linear scales)

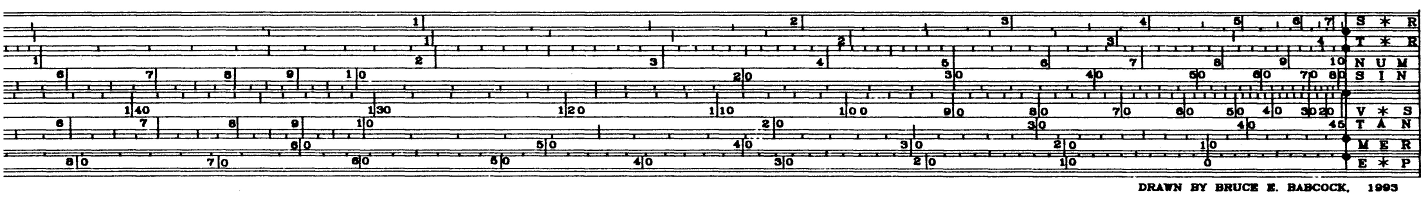
Scale Abbrev.	Full Name	Meaning
	Diagonal Scale on left side	To set exact lengths with dividers, in hundredths of inches or of half-inches
	Inches	Measurement scale of 24 inches along the upper side of the rule
LEA	Leagues	Linear scale for drawing constructions in sea distances: 1 (English) league = 3 sea miles
L and P	Equal Parts, for reading functions of other scales	P for reading RUM, CHO, SIN, TAN, S*T and MER; L for reading the longer RUM & CHO at the very right

Front (goniometric scales)



Scale Abbrev.	Full Name	Meaning	Formula
R U M	Chords of Rhumbs (2 scales)	Chord is twice the half-sine, for compass points (32 in 360°)	$2\sin(5.625 \cdot X)$
C H O	Chords of Degrees (2 scales)	Chord is twice the half-sine, for degrees (360°)	$2\sin(X/2)$
S I N	Sine of Degrees	Sin for degrees (360°)	$\sin(X)$
S E C	Secant of Degrees	Sec for degrees (360°)	$\sec(X) - \sin(90)$
T A N	Tangent of Degrees	Tan for degrees (360°)	$\tan(X)$
S * T	Semi – Tangent	Half-Tan for degrees (360°)	$\tan(X/2)$
M * L or L O N	Miles of Longitude	Length of 1 degree longitude at latitude X	$60\cos(X)$, to be combined with the underlying CHO scale

Back Scales



Scale Abbrev.	Full Name	Meaning	Formula
S * R	(Artificial) Sine of Rhumbs	Logsin for compass points	$\log(\sin(11.25X))$
T * R	(Artificial) Tangens of Rhumbs	Logtan for compass points	$\log(\tan(11.25X))$
N U M	(Artificial) Line of Numbers	2-cycle Log Scale (like the A or B scale of a modern slide rule)	$\log(X)$
S I N	(Artificial) Sine of Degrees	Logsin for degrees (360°)	$\log(\sin(X))$
V * S	(Artificial) VerSine of Degrees	Logversin for degrees (360°)	$\log(1 - haversine(X))$
T A N	(Artificial) Tangent of Degrees	Logtan for degrees (360°)	$\log(\tan(X))$
M E R	Meridional Line	"Waxing of the Latitude" on a meridian of the Mercator map	$\int \sec(X)dX$, to be combined with E*P
E * P	Equal Parts	Linear scale	X

Use with Dividers or Compasses

One cycle on the NUM scale of the Standard Gunter Rule has a length of 11.25 inches. To fit distances up to that value between the tips of the dividers, it would require dividers of at least 8 inches (for a maximum opening angle of some 90°). Sea chart dividers had that size, and were sometimes even larger.

Some Observations on the Scales

The Gunter Rule has different scale characteristics compared with a normal slide rule because of its use with dividers. The modern slide rule has its scales vertically aligned for usage with cursor and vertical hairline. Even on the sector the scales were aligned to the rotation point. On the Gunter Rule the scales can be positioned anywhere as long as the dividers can measure a length on it. Therefore one sees sometimes up to three different scales adjoined on one horizontal line. The lengths of different scales have to correspond, of course: for example, all the logarithmic scales on the back side can be used against one another, to allow any multiplication or division between numbers and goniometrical functions.

Linear Scales, Equal Parts

The linear scales can be recognized by a finer divided forerunner, in front of the real scale: this is the case with the diagonal table and the scales LEA, L, P, and E*P. The full-length "ruler" scale of 24 inches does not have this forerunner.

The forerunner is used for taking more precise distances between the divider points. All these scales are divided by "equal parts", although only the E*P scale has that particular title. These linear scales are based on division in inches or half-inches, with the exception of the E*P scale. The E*P scale is related by a factor 0.6 to the P scale, and it has its forerunner on the MER scale because of their interdependence.

LEA is a half-inch scale factored to cover 200 leagues (600 sea-miles) for use in drawings of sea distances. L and P are radius "unit" scales, used with the left-sided goniometric scales (P, with radius 2 inches), or with the right-sided ones (L, with radius 3 inches), respectively. See also the section on SIN, SEC, TAN and S*T. These scales are sometimes not labeled with "L" and "P", but can be recognized by their position just to the right of the RUM and CHO scales, respectively.

It is interesting to note that all scales on the Standard Gunter Rule, whether in inches or in degrees, have decimal subdivisions: this would have been highly appreciated by Edmund Gunter, who always was a keen advocate of the decimal system.

Chords – CHO

One of the popular scales was the chord scale, which could be used for measuring or constructing angles. The chord "k" of an angle a (AMB) is the line that connects A and B, the points of intersection between the legs of angle a and the circle. The length can be expressed as a goniometrical function: Chord(a) $= 2 \times r \times sin(a/2)$. This explains the old term (double) "halfsine" for the chord function.

An angle of 60° has a chord equal to the radius r.

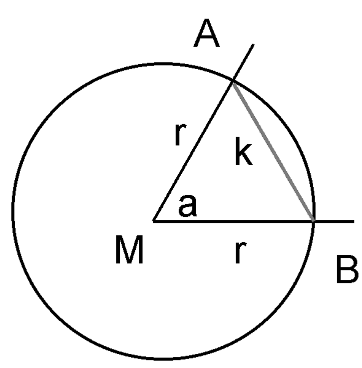


Figure 8. Chord construction.

To construct a given angle, first the radius r is taken with compasses from the chord scale at 60° (frequently used values like this one have a brass inlay on the scale to fixate the points of the compasses and to protect the surface of the rule). Then the circle with this radius is drawn. Finally the chord length of the required angle is copied from the chord scale to the circle drawn.

Rhumbs – RUM

Mariners in Gunter's time already used the magnetic compass with an angular scale called "rosa".

The compass rose was (and still is) divided into 32 compass points, also called rhumbs. While the main compass points are North, East, South, and West, the intermediate compass points have logical names like North-West, East-North-East, etc.

One rhumb is equivalent to 360 / 32 = 11.25 degrees.

Some goniometric functions like chords and artificial sines and tangents had separate scales on the Standard Gunter Rule, one for degrees and one for rhumbs, thereby proving its maritime nature. Those double scales were vertically aligned, with eight compass points over the 90° mark.

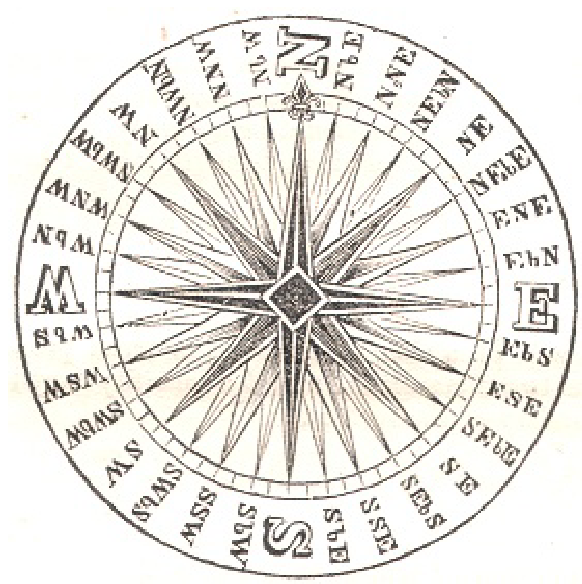


Figure 9. Rhumb compass points.

The scale called RUM is actually a chord scale, but for rhumb compass points instead of degrees. The Standard Gunter Rule even has two sets of CHO/RUM scales, for different radii (2 and 3 inches, from 0 to the radius 60).

Goniometrical scales – SIN, SEC, TAN and S*T

These goniometric scales are non-logarithmic, and can be used in more than one way. In the first place, they can be used to determine the value of the goniometric scale, like a visual sine-table. For this purpose the distance between zero and the required angle on, say, the sine scale, is transferred with the dividers to the unit scale P, where the sine of that angle can be read on the equal parts divisions. The other use could be trigonometric calculations by constructing a drawing on chart or paper, which mimics the working of the Sector by congruent triangles. The secant scale (not always labelled as such) is a continuation of the sine scale, as can be seen in the constructional drawings of the time (see Bion, [11], Plate IV, fig. 5). The value of a secant should therefore be taken from the start of the sine scale.

The "semi-tangent" tan (X/2) was used in stereographic projections, where angles from the center of a sphere needed to be replaced by half-angles from the projection point at the opposite end of the sphere.

Miles of Longitude – M * L or LON

On the equator of the earth, the arc of one degree longitude covers 60 nautical miles. For higher latitudes this number decreases with the cosine of the latitude. The number of nautical miles in one degree longitude of a parallel are read on the M * L scale against the latitude of that parallel on the underlying CHO scale. The result on M * L ranges from 60 on the equator to 0 on a pole.

Logarithmic Scales - NUM, SIN, TAN, S * R, T * R, V * S

At figure 5, the NUM, SIN, and TAN have already been addressed. Just like the CHO scale, also the artificial SIN and TAN scales have their nautical counterparts, with angles expressed in compass points: the "Sines of Rhumbs" (S * R) and the "Tangents of Rhumbs" (T * R) scales. The scale title V * S means "versed sines". but it does not reflect our current versine which equals (1 - cosine). It actually represents, in modern terminology, the logarithm of (1 - haversine), or $\frac{1}{2}$ (1 + cosine).

The haversine function (half-versed-sine) was introduced for computational reasons in the formula that calculates the angular distance between two arbitrary points on the globe. This formula could be calculated between the NUM, the SIN and the V * S scales on the Gunter rule.

The Meridional Scale – MER

In the late 16th century, the Mercator map projection created the possibility for navigators to plot compass courses as straight lines. These maps were called "waxing the latitude", because for every additional degree latitude the vertical scale of the map was increased proportionally with the horizontal scale which kept the meridians as parallel lines. The increment to the vertical scale at a certain point was proportional to the secant of its latitude, and in a Mercator map the vertical distance of a given latitude from the equator was composed of the sum of all these increments. With current analytical methods we know that this function can be derived from the integral of the secant function, resulting in the "inverse Gudermannian". But in the times of the Gunter rule, Meridional tables had been constructed by successively incrementing the secants along the whole range of latitudes.

The Meridional scale is used in combination with the E * P scale: in the first 10 degrees the scales coincide because the "waxing of the latitude" is not yet visible. For higher values of latitude on the M E R scale, the increased distance from the equator can be read on the E * P scale.

In calculations it is sometimes needed to express the "waxed latitude" in sea-miles; then the required latitude on the M E R scale can be read on the P scale (using the factor 0.6), on the other side of the rule. If the distance on the M E R scale is larger than the P scale unit value, it is required to "somersault" the dividers, with the P scale unit between the tips, n times over the latitude, and then read the remainder of the latitude on the P scale again. The "waxed miles" are then: 100 x (10 x n + (the last P scale reading)).

Accuracy of Scales

In principle, the large size of the Gunter Rule would allow for good accuracy, but the production methods used resulted in an accuracy much worse than, say, of a large desktop Nestler.

First, the makers economised on the scale divisions: on the NUM scale some 130 division marks were made, while a modern 50 cm A-scale can have three times as many. On the SIN scale the distance between two marks can be more than 1 cm, which makes visual interpolation difficult. Second, the divisions were made by hand, before the introduction of the "dividing" engine. This added to the inaccuracy: it has been reported, see page 29 in [25], that Gunter rules could be engraved by hand with a speed of about one mark per second. This would bring the total mark engraving time of a Standard Gunter Rule, with more than 1200 marks in all, to a full 20 minutes of concentrated and error-prone work. Lastly, the scale designs were being copied and copied again, sometimes including mistakes. For example, my own collection includes a 1-foot Gunter Rule where the RUM and CHO scales differ more than 5% in length! One can get a quick impression of the precision of a Gunter rule by "somersaulting" the dividers along the marks of one of the linear scales. For example, the L and P scale divisions can be so imprecise that at first I thought they represented a function, until measurements revealed random deviations from a linear scale. Also, the total length of the L scale especially can vary widely from the nominal 3 inches for different rules.

On the other hand, the mariner did not always need the highest accuracy. He measured his course not in degrees but in compass points, and that was the best accuracy with which he could steer under the conditions of strong waves, or inaccurate magnetic deviation and variation.

Some of the most frequent calculations made, were those in "dead reckoning" (strange name, derived from "deduced" reckoning), where a new position was deduced from a previous position in a "course triangle" consisting of speed vectors for course steered, side effect of current and wind, and the true course as result. This deduction was either done by construction with compasses on chart or paper, or by calculation with the Gunter rule on NUM and SIN/TAN scales (again this Rule of Sines).

Example from Bowditch

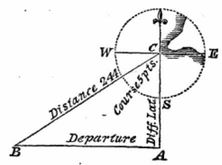
From the 1851 edition of Bowditch, [13], the most simple example will be shown for "plain sailing", i.e., not yet taking into account the effects of a Mercator type chart. The three possible methods of solution of the problem are given: the construction by projection (drawing), calculation by logarithmic tables, and solution by Gunter's scales.

Course and distance sailed given, to find the difference of latitude and departure from the meridian.

A ship from the latitude of 49° 57′ N., sails S. W. by W. 244 miles; required the latitude she is in, and her departure from the meridian sailed from.

BY PROJECTION.

Draw the line CA, to represent the meridian of the place C, from whence the ship sailed. With the chord of 60° in your compasses, and one foot in C, as a centre, describe the compass W. S. E. Take 5 points in your compasses from the line of rhumbs on the plane scale, and set it off on the arc, from S. towards W., for the course; through this point and C draw the line CB, and make it equal to the distance 244; draw BA parallel to the east and west line EW, to cut the meridian in A. Then will CA be the difference of latitude 135.6, and AB the departure 202.9.



BY LOGARITHMS.

By making the distance radius.

To find the departure.		To find the difference of latitude.	
As radius 8 points............	10.00000	As radius 8 points............	10.00000
Is to the distance 244	2.38739	Is to the distance 244	2.38739
So is the sine course 5 points ..	9.91985	So is the cosine course 5 points.	9.74474
To the departure 202.9........	2.30724	To the difference of lat. 135.6..	2.13213

Now, as the ship is in north latitude sailing southerly,

From the latitude left	49° 57′ N.
Take the difference of latitude 135.6....................	2 16 S.
Gives the latitude in..................................	47 · 41 N.

And the departure from the meridian is 202.9 miles.

BY GUNTER.

Extend from radius or 8 points * to 5 points on the line marked SR; that extent will reach from the distance 244, to the departure 202.9, on the line of numbers.

2dly. Extend from radius or 8 points to 3 points, the complement of the course, on the line SR; that extent will reach from the distance 244, to the difference of latitude 135.6, on the line of numbers.

Thus may all the operations be performed in the several cases of Navigation.

Other Rules for Navigation

The Standard Gunter Rule, although widely known to collectors, was certainly not the only rule with scales designed for navigation. Around the same time that Gunter published his work on the cross-staff, etc., a small book, see [6], was published by John Aspley, titled *Speculum Nauticum ...* .

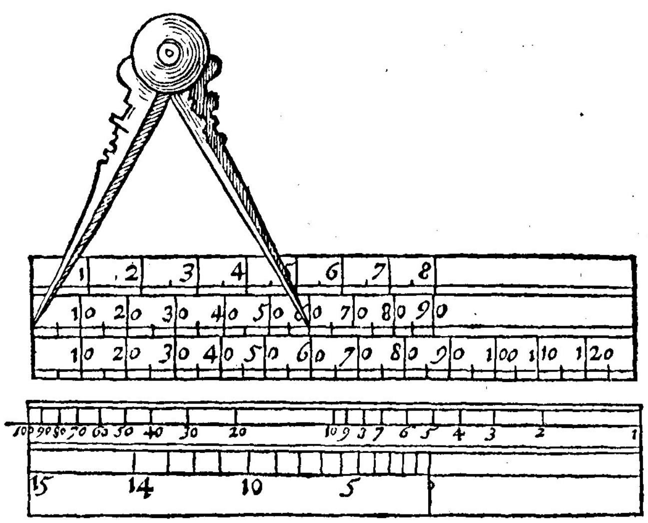


Figure 10. Aspley's Plain-Scale.

This book explains the use of a so-called "Plain-Scale", a rule with only 5 scales, which are the equivalents of the Gunter scales RUM, CHO, E*P on the front, and NUM and M*L on the back.

The Plain-Scale or Plane-Scale, was the general name of a navigation rule designed to project spherical problems onto the 2-dimensional plane of chart or drawing. Plains-Scales are called Gunter rules, when at least the NUM scale is included in the set of scales.

Aspley only gives a picture of the front side, but in reference [16] the Dutch expert on the history of navigation, Ernst Crone, supplies a picture of front and back of the Plain-Scale. This more complete picture has been found in a Dutch handbook for navigators [8] which addresses the same subject as Aspley's book, though without any reference to him. But even Aspley may not have been the original Plain-Scale inventor, who must have been well aware of NUM, Gunter's "Line of Numbers".

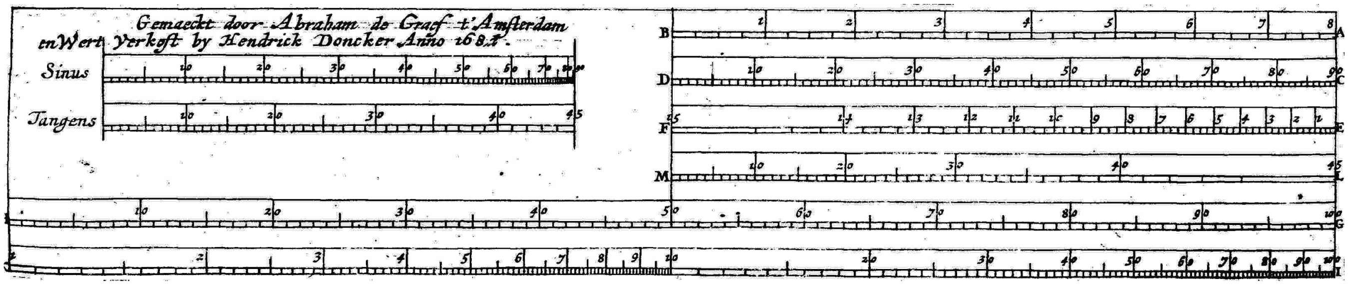
This Plain-Scale picture is remarkable in that the 2-cycle logarithmical scale on the back is an inverse scale, and that the M*L scale is different in that it is divided into 15 units of 4 sea miles each ("German" miles); in Aspley's version the M*L scale is divided into 20 leagues (of 3 sea miles each).

Another "Plain-Scale" has been described in [9], one of the Dutch navigation handbooks of the mid-17th century. Abraham de Graef describes a somewhat extended Plain-Scale (one -sided) where the table in Figure 11 translates his scale indications into our usual Gunter scale names.

Actual Usage of Gunter Rules or Plain-Scales

Looking in navigation handbooks, we can gain some insight into which type of navigation rule was used, by country. Obviously, the Gunter rule was used in England, but also in the United States because it is mentioned far into the 19th century in the handbook [13] of Bowditch: *The New American Practical Navigator.* In Germany, at least one navigation handbook [14], by Jerrmann, 1888, described the use of the Gunter rule.

In Holland, the Gunter rule may have been used less, or maybe not at all. In 1624 Edmund Wingate had published a book [5] & [15], describing Gunter's scales (while giving him proper credit), not long after Gunter's own publication. This book was translated into Dutch, see [7], including a fold-out picture of the three Gunter scales. Consequently, a description of Gunter's scales was easily available in the Netherlands, via this detour. A number of Dutch navigation handbooks from the 17th century have been browsed, but none of them mention the Standard Gunter Rule: only some simple versions of a Plain-Scale are described. But then, one would expect more of these Plain-Scales preserved or recovered, just as so many Standard Gunter Rules have survived.



de Graefs scale names	Gunters scale names
Sinus	S I N (non-log)
Tangens	T A N (non-log)
B	R U M
D	C H O
F	M * L
M	T A N (non-log)
	E * P
	N U M

Figure 11. de Graef's Plain-Scale.

In a series of articles [17] and [18], a description is given of one Plain-Scale that was found in the Dutch East Indiaman sailing vessel "Hollandia", wrecked in 1743 off the Scilly Isles (see figure 12).

That particular Plain-Scale had a different design, with scales on one side and a diagonal table on the other. The table on the left gives the relation to our usual Gunter scale names.

The Hollandia Plain-Scale is signed by Johannes van Keulen (JVK), a well-known cartographer and instrument maker for the VOC ("Verenigde Oost-Indische Compagnie", or Dutch United East India Company). This object is part of the collection in the "Rijksmuseum" in Amsterdam.

Hollandia scale names	Gunters scale names
V (for "Uren")	Chord scale, but expressed in Hour Angles (24 in 360)
H (for "Hoekmaat")	S I N (non-logarithmic)
S (for "Streken")	R U M
C (for "Coorden")	C H O
L N (for "Linea Numerorum")	N U M
V	V (smaller radius)
S	S (smaller radius)

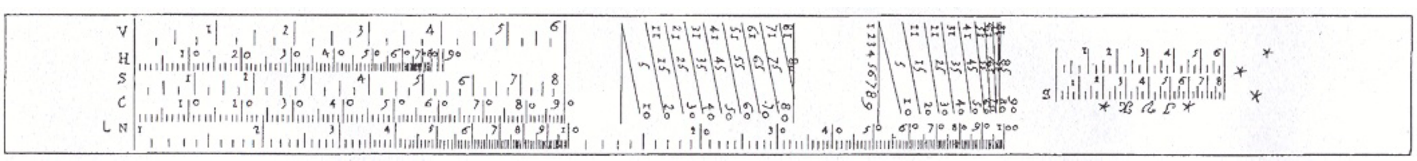


Figure 12. The Hollandia Plain-Scale

Only three other specimens of a Plain-Scale are known in the Netherlands.

One is in the "Nederlands Scheepvaartmuseum" in Amsterdam, signed by Jacobus Kley, comparable to the Hollandia Plain-Scale.

Another is in the "Zeeuws maritiem muZEEum" in Vlissingen, with the striking characteristic that it does NOT have the logarithmic Line of Numbers!

The scales are (see figure 13):

- M ("Mijlen") is an equal parts scale.
- S ("Sinus") is the non-logarithmic sine scale.
- L ("Lengte") is the Longitude scale (M*L on the Gunter rule).
- C ("Coorde") is the Chord scale.

The maker *IS* is probably Isaak Swigters from "In de Jonge Lootsman", Amsterdam. This Plain-Scale was recovered from the VOC ship " 't Vliegend Hart ", that was lost on the shoals west of Walcheren in 1735, one of the more than 100 wrecks around the Westerschelde estuary of Vlissingen.

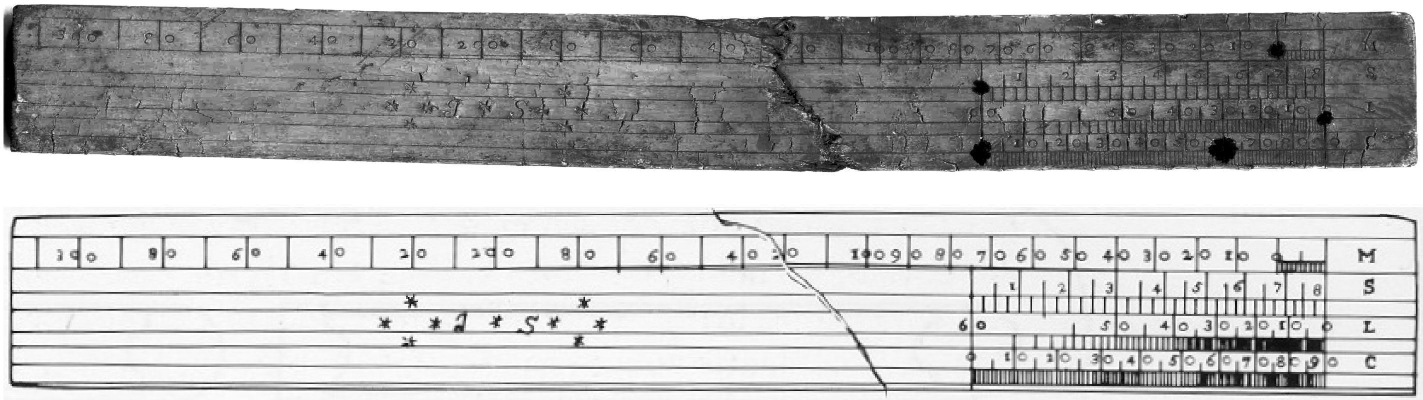


Figure 13. The Plain-Scale of 't Vliegend Hart.

The last one (maker unknown) is owned by the "Museum Boerhaave" in Leiden (see figure 14). It consists purely of diagonal scales for only two functions: the sine (H="Hoekmaat") and the chord (C="Coorde"), for three different scale factors. This specimen is so limited in functions that we hardly dare call it a Plain-Scale.

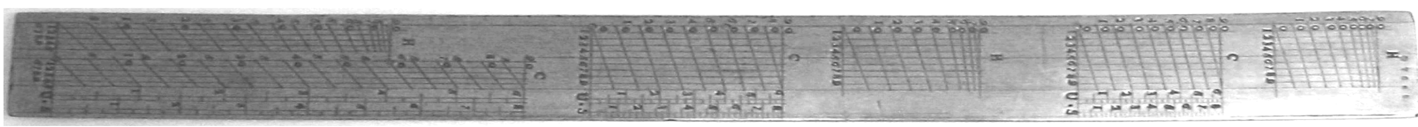


Figure 14. The Boerhaave Plain-Scale.

Another source of information consists of actual lists of navigation equipment supplied. In [19] a list is discussed in which the standard navigating equipment for the VOC ships is named, with prices and numbers used per ship. In 1747, three "Pleinschalen" (Plain-Scales) were required per ship, at a price of 12 "Stuyvers" (or 0.60 gulden) each: this may be compared with the more than 100-fold price of a modern Hadley octant (75 gulden). Also in other equipment lists only "Pleinschalen" are mentioned, and not Gunter rules. On the other hand, Gunter rules were actually mentioned in catalogs of Dutch instrument makers, for example Johannes van Keulen lists in 1777 a Gunter rule for 1.50 gulden.

Conclusions

The Gunter rules, that most collectors know and seek, are so identical to each other that we call them Standard Gunter Rules. There are variations, sometimes with small differences, but in the Netherlands some versions of the general Plain-Scale appear to have had preference over the Standard Gunter Rule. Gunter rules have by definition one or more logarithmic scales. A number of other Plain-Scales are known without any logarithmic scale at all. The non-logarithmic scales on Gunter rules and Plain-Scales are mostly derived from earlier instruments, like the sector and the cross-staff. It appears that Gunter was not the designer of the complete Standard Gunter Rule, although he is conclusively considered to be the inventor of the three basic logarithmic scales NUM, SIN and TAN. This is emphasized by Oughtred himself, when he writes modestly about his own Circles of Proportion, see [12]:

> *"For these, I must freely confess, I have not so good a claim against all men, as for my Horizontal Instrument*[2]*. The honour of the invention [of logarithms] next to the lord of Merchiston, and our master Briggs, belonging (if I have not been wrongly informed) to master Gunter, who exposed their numbers onto a streight line. And what doth this new instrument, called the Circles of proportion, but only bowe and inflect master Gunter's line or rule."*

Further study is needed to discover the real roots and the development line of the Standard Gunter Rule.

Acknowledgments

Many fellow collectors have contributed to this paper by providing information and pictures of their own Gunter rules, or with remarks and suggestions, an example being the very stimulating discussions with Rob van Gent and Cyron Lawson on the more exotic scales of the Gunter rule.

The assistance of Mörzer Bruyns of the "Nederlands Scheepvaartmuseum" in Amsterdam, who reviewed the paper on navigational aspects, is especially appreciated.

[2]See the inside front cover of the Fall 2003 *JOS* for a picture of Oughtred's horizontal instrument. *ed.*

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Miscellaneous Rules – III

Bob Otnes

The example below is unmarked. The back is blank. It is a Soho or Engineer's type rule, probably made in Great Britain. It is almost like new. I am not able to give anything but the roughest estimate of age: 1850 to 1940. I am not going back earlier than 1850 mainly on the basis of condition. On the other hand, it could have been made in Birmingham last year.

The most interesting part about it is the length of the scales: eleven inches! Its shape compares to that of so-called "proof rules" that were popular in Britain, noting that most of those had two slides and had scales on both sides.